Profesor:
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PITAGORAS
ACADEMIA

ÁLGEBRA

GRUPO PITÁGORAS

RELACIONES

$1.-$*Producto Cartesiano*

Sean A y B dos conjuntos no vacios, se define al producto cartesiano AxB de la siguiente manera:

$$AxB = \{(x;y)/x \in A \wedge y \in B\}$$

Ejemplo:

$$Sea\ A = \{1;2;3\} \ \wedge B = \{7;8;9\}$$

$$AxB = \{(x;y): x \in \{1;2;3\} \wedge y \in \{7;8;9\}\}$$

$$AxB = \{(1;7),(1;8),(1;9),(2;7),(2;8),(2;9),(3;7),(3;8),(3;9)\}$$

$$BxA = \{(x;y): x \in \{7;8;9\} \wedge y \in \{1;2;3\}\}$$

$$AxB = \{(7;1),(7;2),(7;3),(8;1),(8;2),(8;3),(9;1),(9;2),(9;3),\}$$

$2.-Observaciones$

$AxB \neq BxA$ $\quad$ $(es\ decir\ el\ producto\ cartesiano\ no\ es\ conmutativo)$

$n(AxB) = n(A).n(B)$

$n(AxB) = n(BxA)$

$3.-Relación\ de\ A\ a\ B$

$Una\ relación\ de\ A\ a\ B\ es\ un\ subconjunto\ de\ AxB$

$Principalmente\ se\ estudiarán\ las\ relaciones\ de\ A\ a\ B\ definidas\ por\ una\ ley\ de\ formación$

$$R = \{(x;y) \in AxB:\ p(x;y)\}$$

$$Donde\ p(x:y) es\ la\ propiedad\ que\ deben\ cumplir\ x \in A\ \wedge\ y \in B$$

Estos productos cartesianos también se pueden definir para conjuntos infinitos

$$\mathbb{R}x\mathbb{R} = \mathbb{R}^2 = \{(x; y): \ x \in \mathbb{R} \wedge y \in \mathbb{R}\}$$

$$\mathbb{Z}x\mathbb{Z} = \mathbb{Z}^2 = \{(x; y): \ x \in \mathbb{Z} \wedge y \in \mathbb{Z}\}$$

$Por\ ejemplo.\ al\ inicio\ definimos\ AxB\ con\ A = \{1:2:3\} \wedge B = \{7;8;9\}$

$$AxB = \{(1;7),(1;8),(1;9),(2;7),(2;8),(2;9),(3;7),(3;8),(3;9)\}$$

$Podemos\ definir\ a\ la\ relación\ R_1 de\ la\ siguiente\ forma:$

$$R_1 = \{(x;y) \in AxB:\ x + y = 10\}$$

$Y\ de\ esta\ manera, los\ únicos\ elementos\ de\ R_1 serían:$

$$R_1 = \{(1;9),(2;8),(3;7)\}$$

$Otro\ ejemplo:$

$$R_2 = \{(x;y) \in BxA: x^2 > y - 5\}$$

$$R_2 = \{(7;1),(7;2),(7;3),(8;1),(8;2),(8;3),(9;1),(9;2),(9;3),\}$$

Una función es un tipo especial de relación.
Es una relación en la que la primera componente solo se relaciona con un único valor (2da componente)

$A = \{3; 4; 5; 6\}$ $\quad\quad B = \{1; 2; 3; 4]$

$$R_1 = \{(x; y) \in AxB: xy < 12\}$$

$$R_1 = \{(3; 1), (3; 2), (3; 3), (4; 1), (4; 2), (5; 1), (5; 2), (6; 1)\}$$

Es una relación pero no es una función

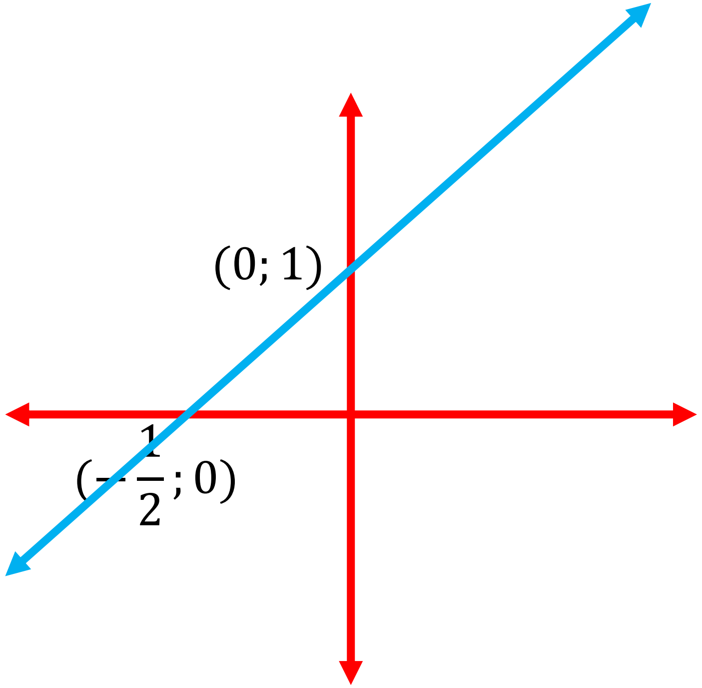
$$R_2 = \{(x; y) \in AxB: x = 2y\}$$

$$R_2 = \{(4; 2), (6; 3)\}$$

Es una relación y también es una función

$Rectas:$

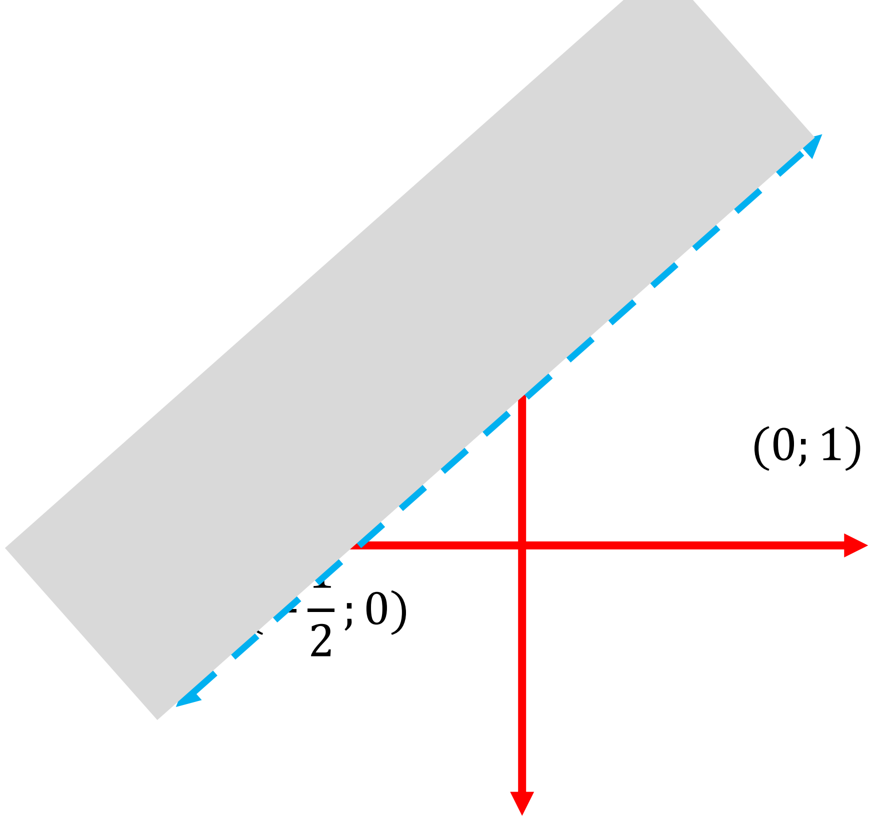
$R_1 = \{(x;y) \in \mathbb{R}^2 : y = 2x + 1\}$



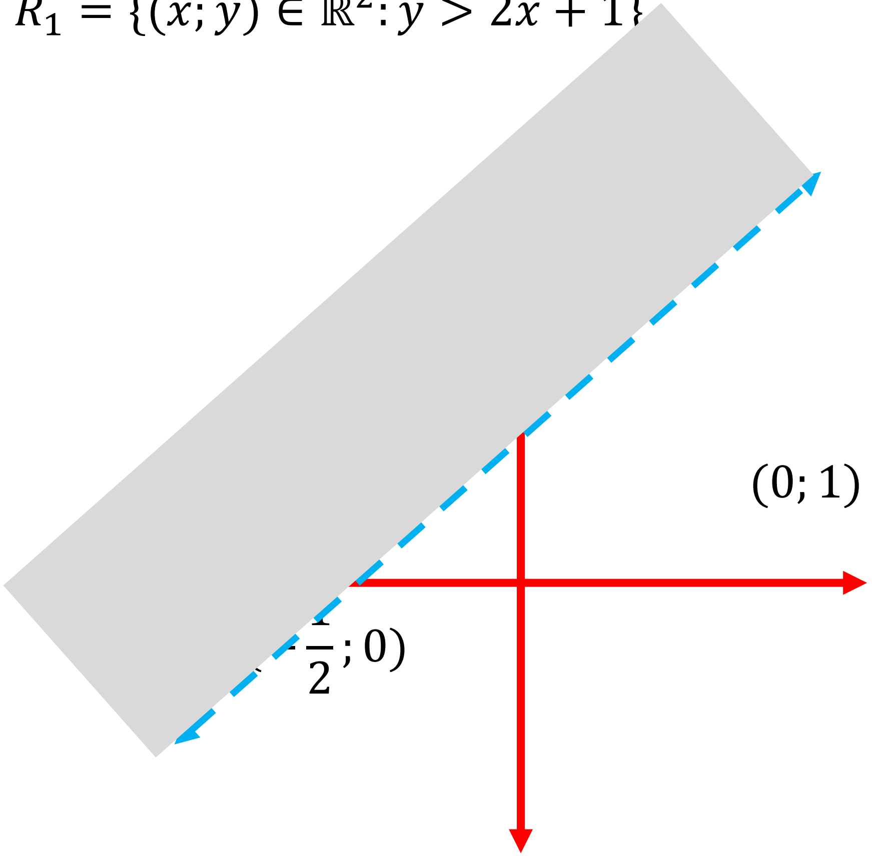
$x = 0 \quad entonces\ y = 1 \quad \ldots.(0;1)$

$x = -\frac{1}{2} \quad entonces\ y = 0 \quad \ldots.(-\frac{1}{2};0)$

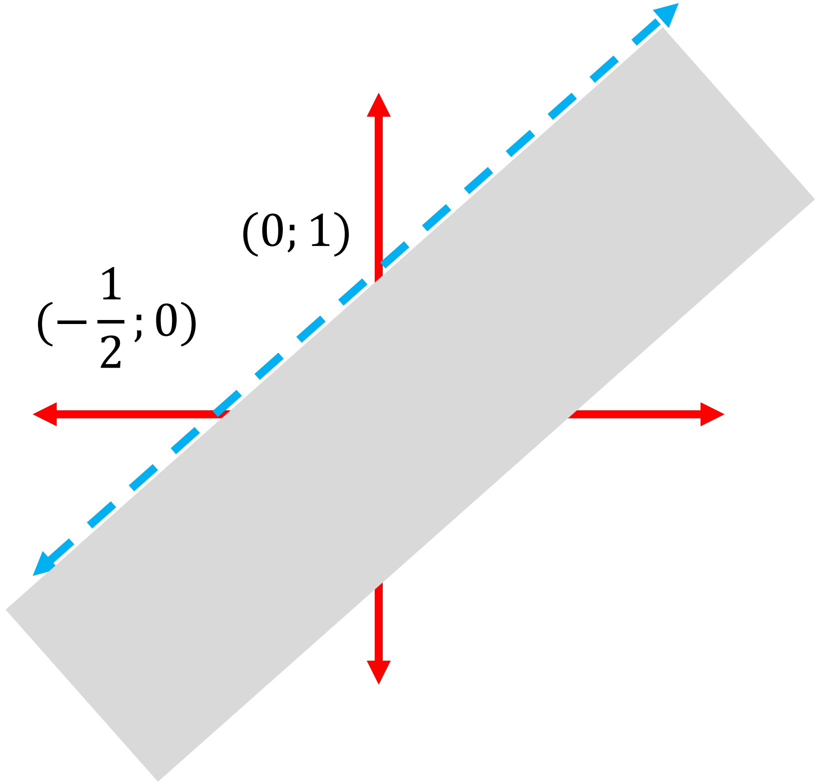
$R_1 = \{(x;y) \in \mathbb{R}^2 : y > 2x + 1\}$



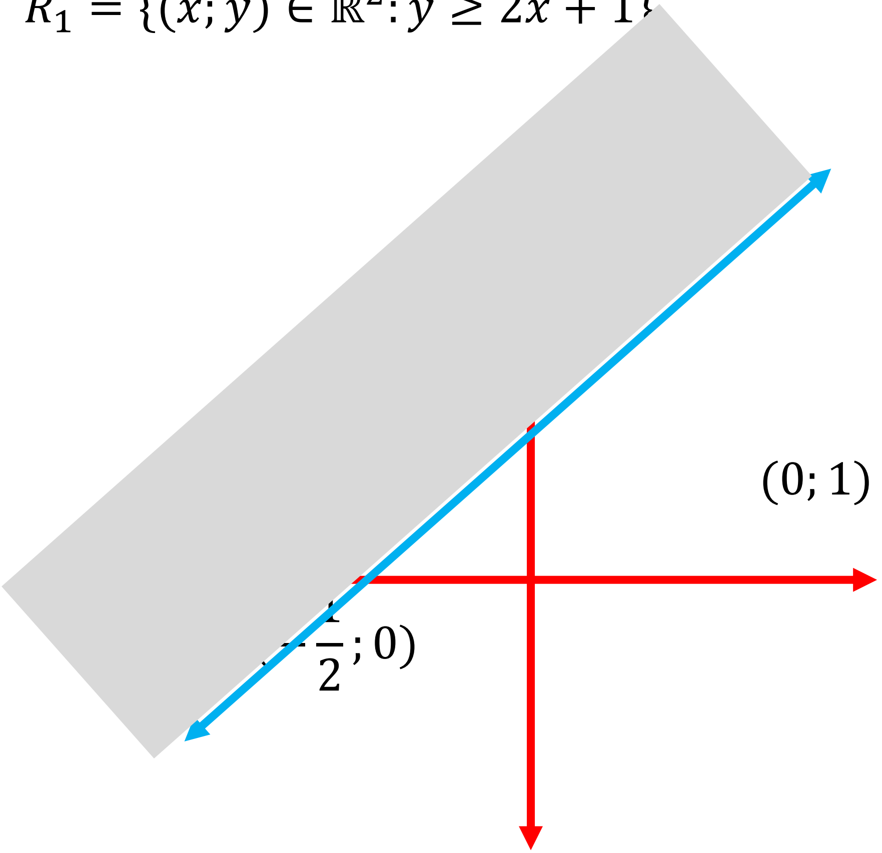
$R_1 = \{(x; y) \in \mathbb{R}^2 : y > 2x + 1\}$



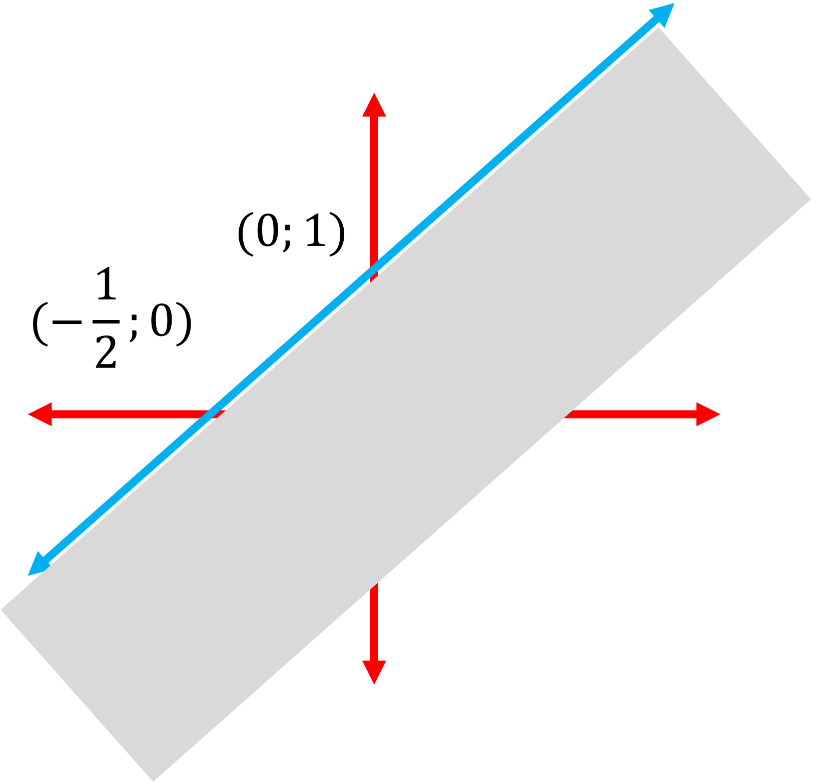
$R_1 = \{(x; y) \in \mathbb{R}^2 : y < 2x + 1\}$



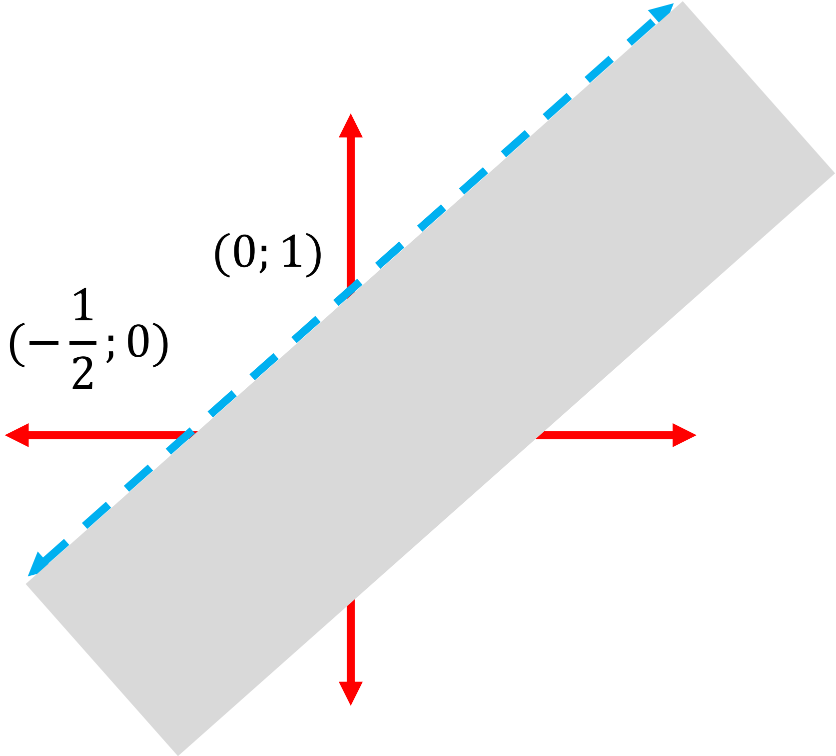
$$R_1 = \{(x;y) \in \mathbb{R}^2 : y \geq 2x + 1\}$$



$$R_1 = \{(x;y) \in \mathbb{R}^2 : y \leq 2x + 1\}$$



$R_1 = \{(x;y) \in \mathbb{R}^2 : y < 2x + 1\}$

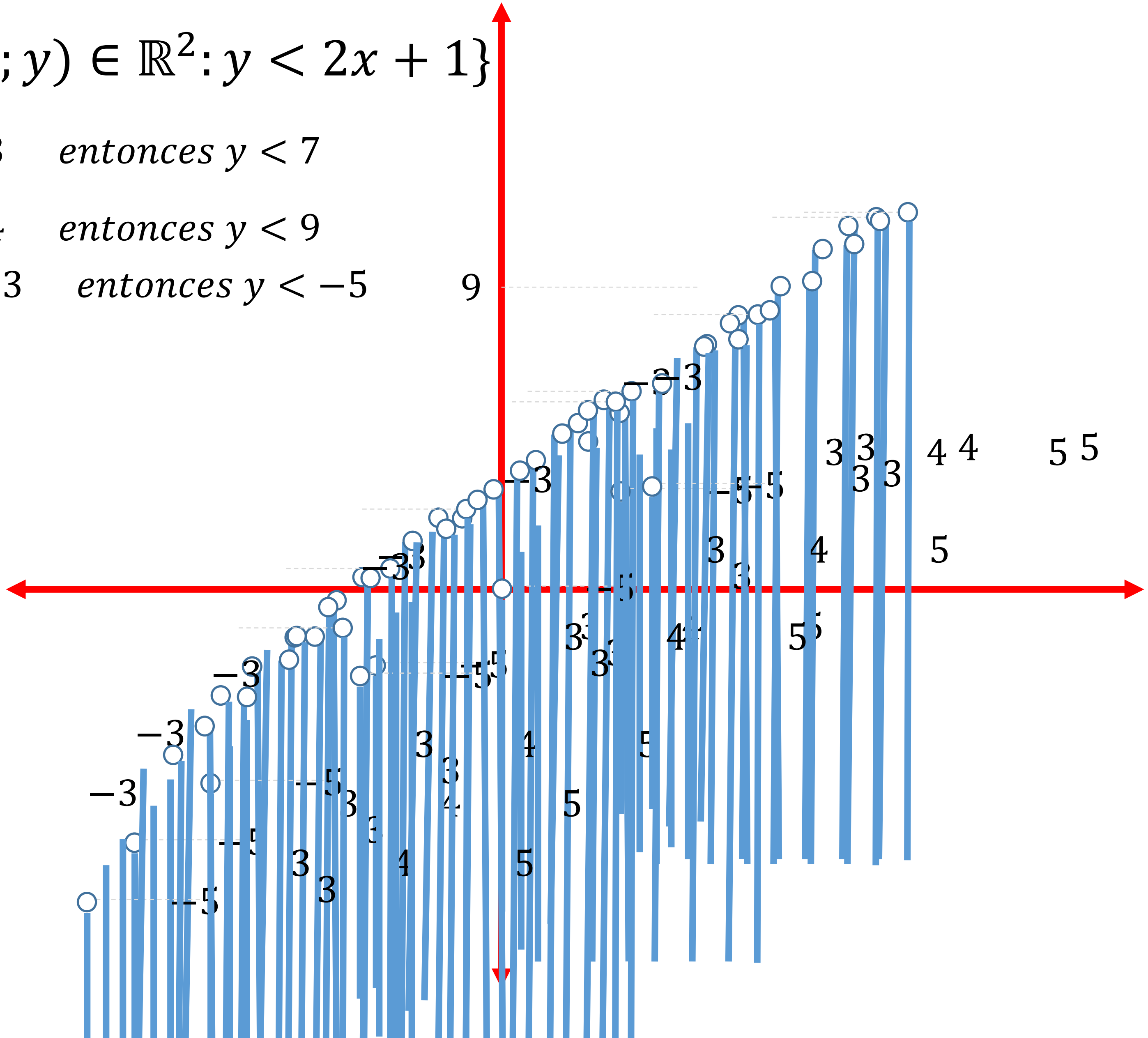


$$R_1 = \{(x;y) \in \mathbb{R}^2 : y < 2x + 1\}$$

$x = 3 \quad entonces\ y < 7$

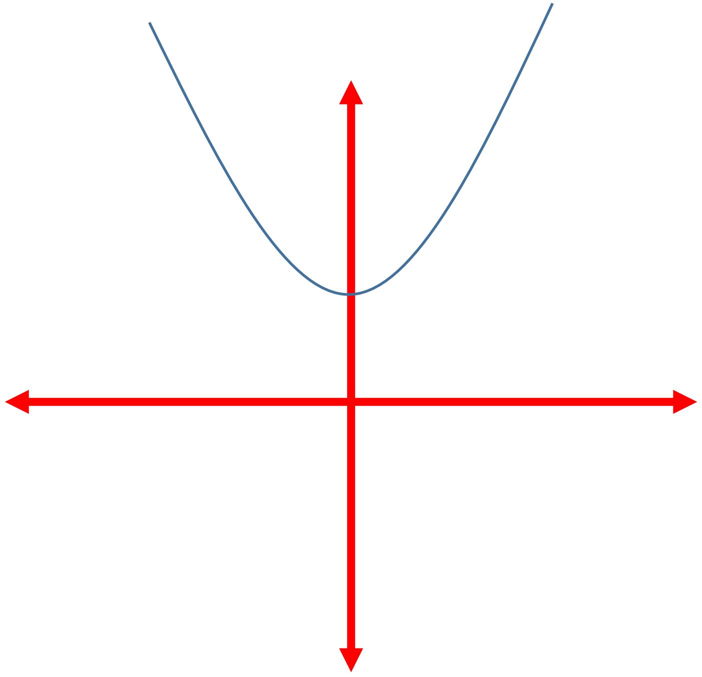
$x = 4 \quad entonces\ y < 9$

$x = -3 \quad entonces\ y < -5$

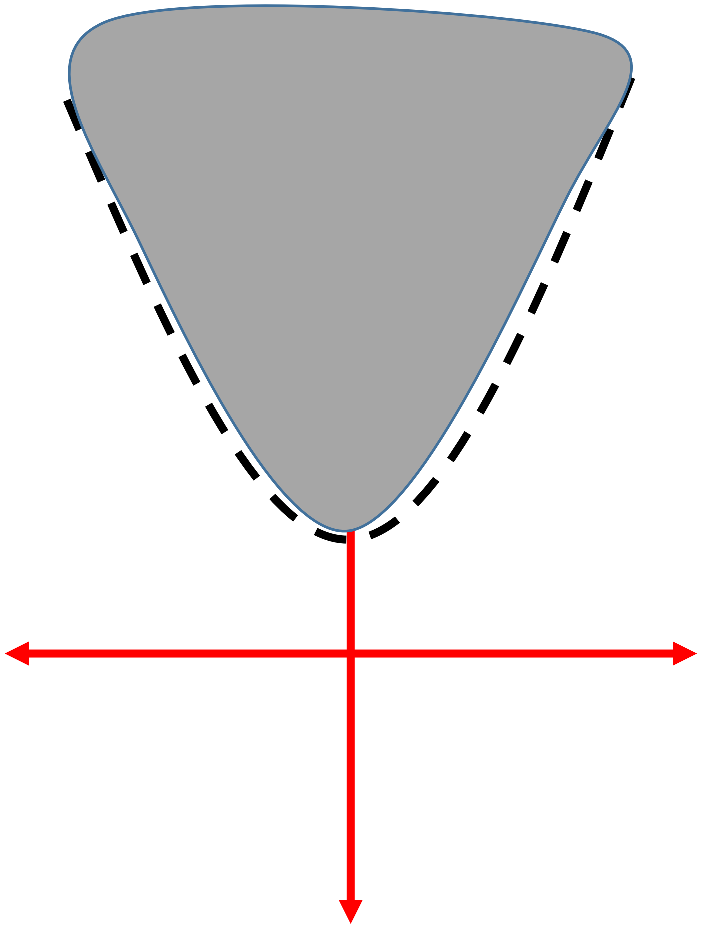


Parábolas

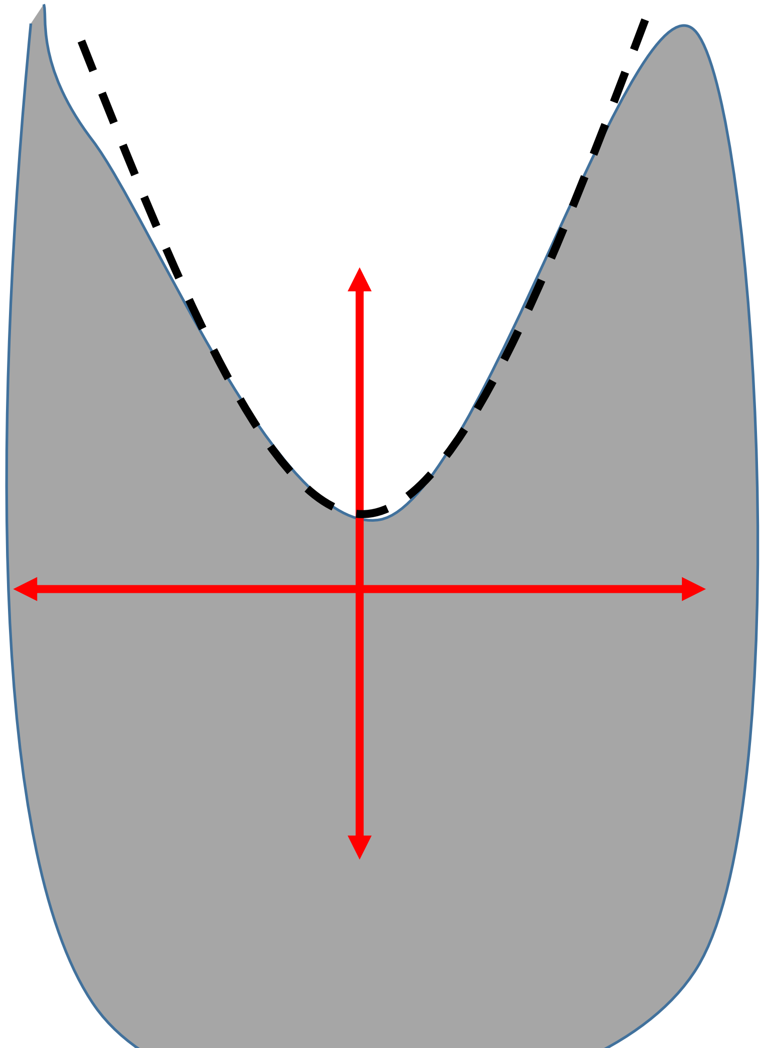
$R_1 = \{(x; y) \in \mathbb{R}^2 : y = x^2 + 1\}$



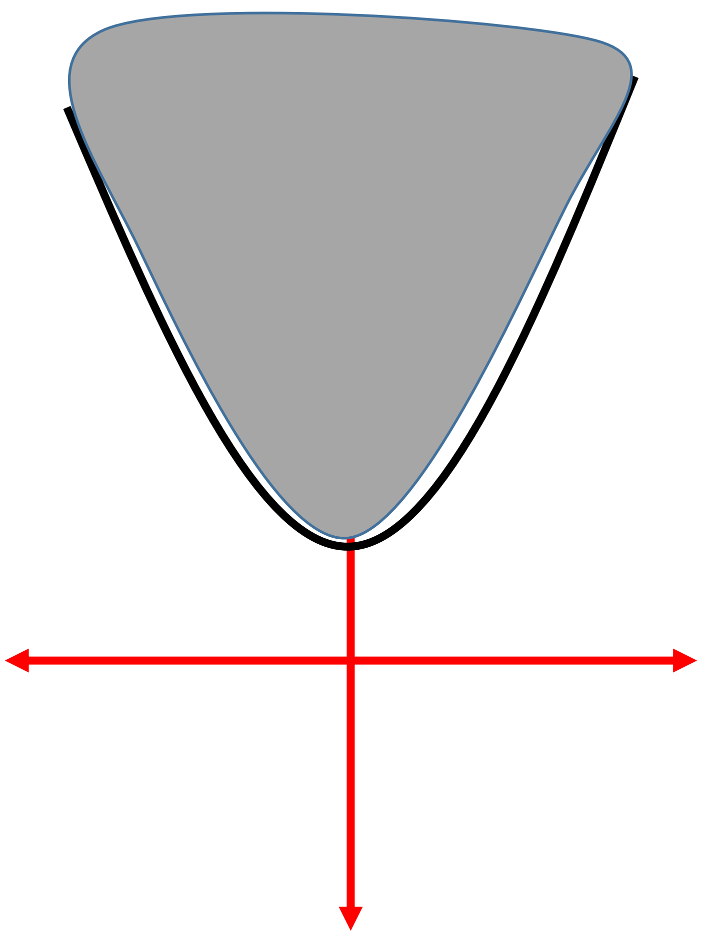
$R_1 = \{(x; y) \in \mathbb{R}^2 : y > x^2 + 1\}$



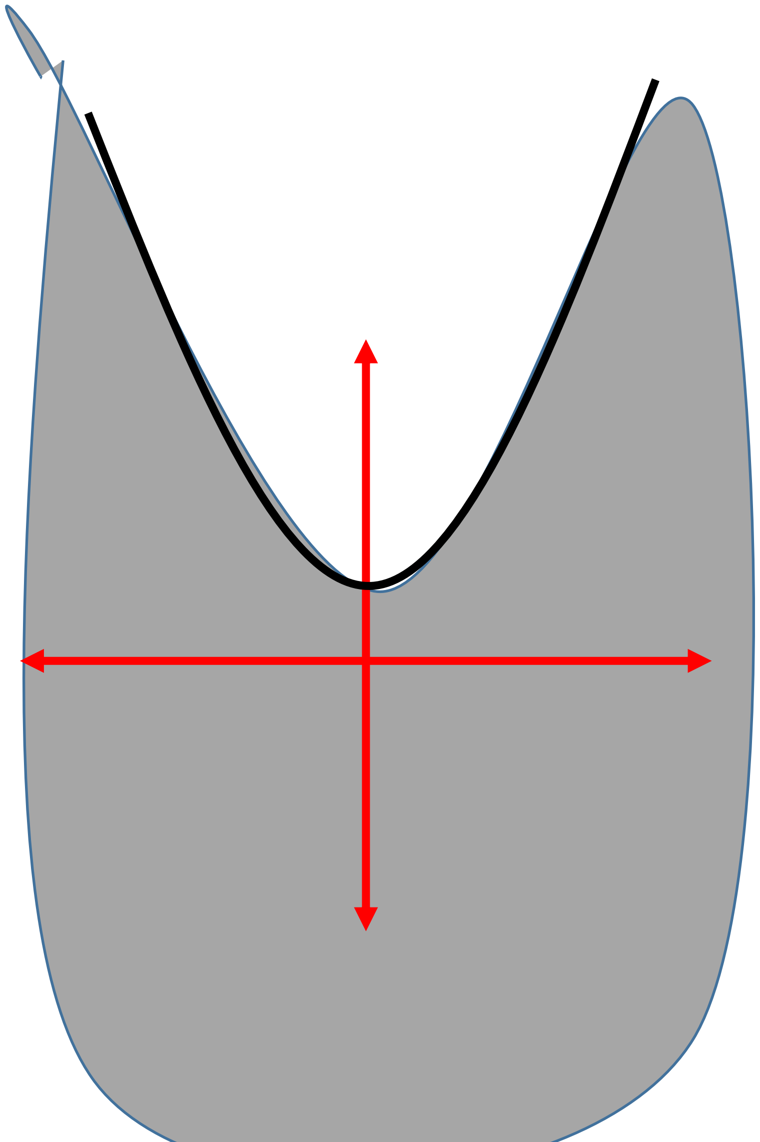
$R_1 = \{(x; y) \in \mathbb{R}^2 : y < x^2 + 1\}$



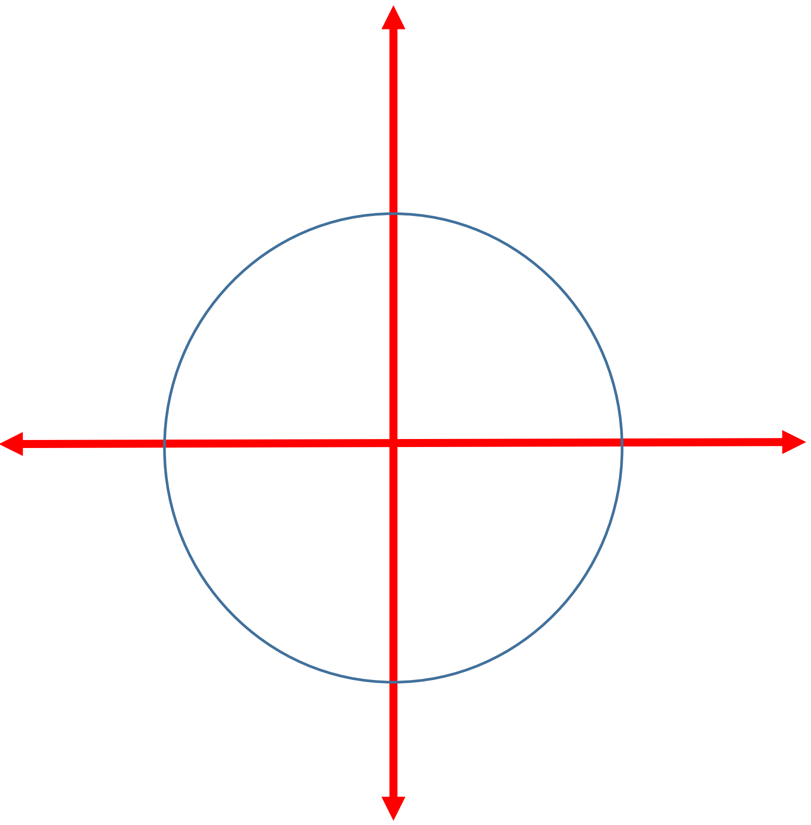
$R_1 = \{(x; y) \in \mathbb{R}^2 : y \geq x^2 + 1\}$



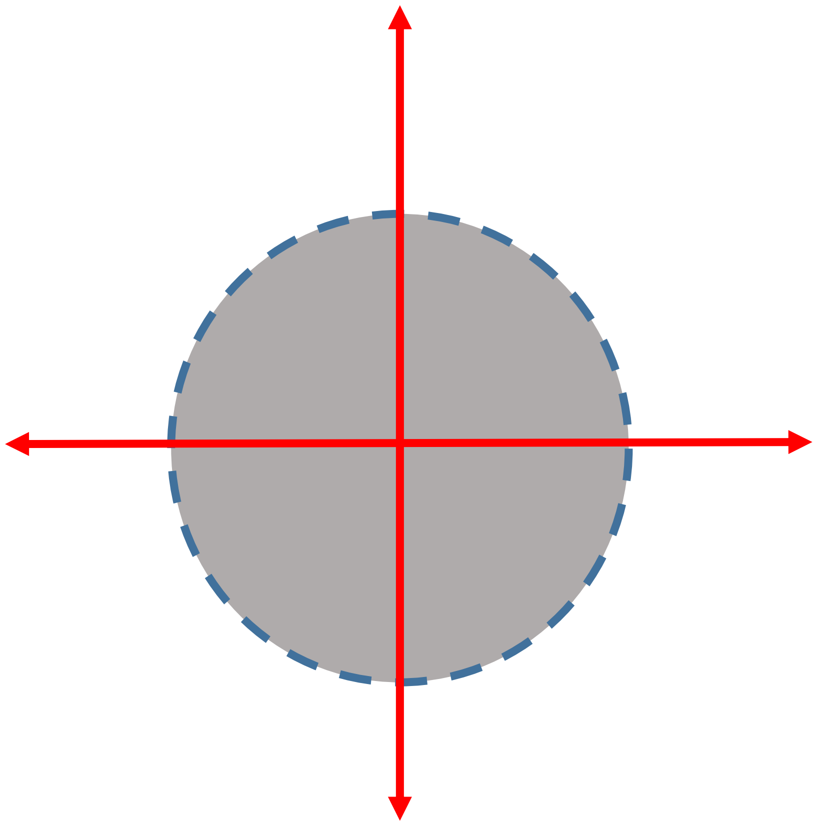
$R_1 = \{(x; y) \in \mathbb{R}^2 : y \leq x^2 + 1\}$



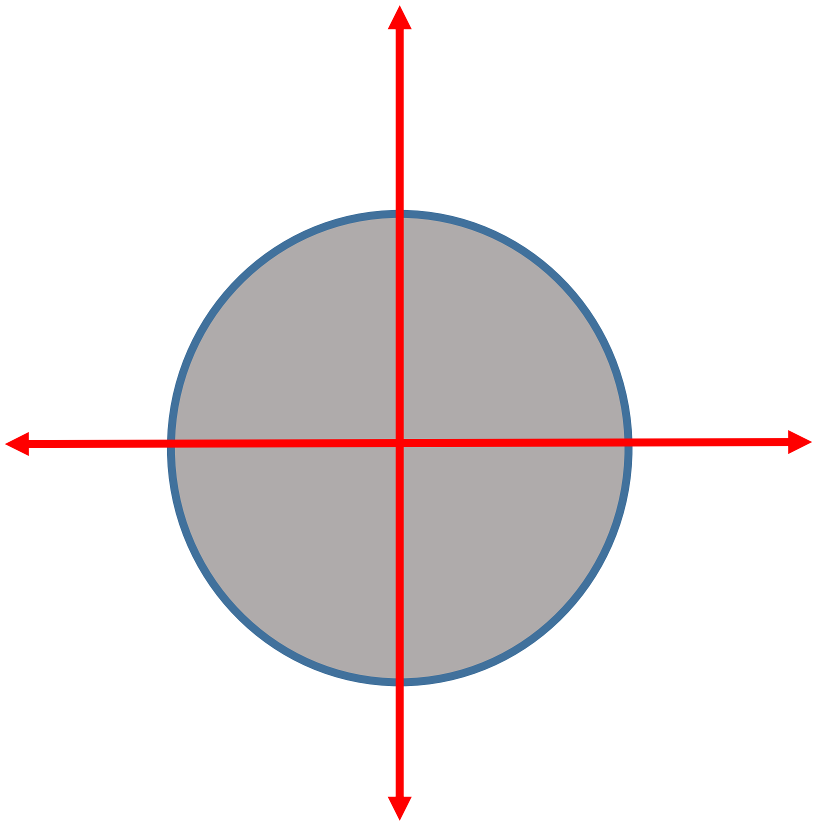
$R_1 = \{(x;y) \in \mathbb{R}^2 : x^2 + y^2 = 25\}$



$R_2 = \{(x;y) \in \mathbb{R}^2 : x^2 + y^2 < 25\}$



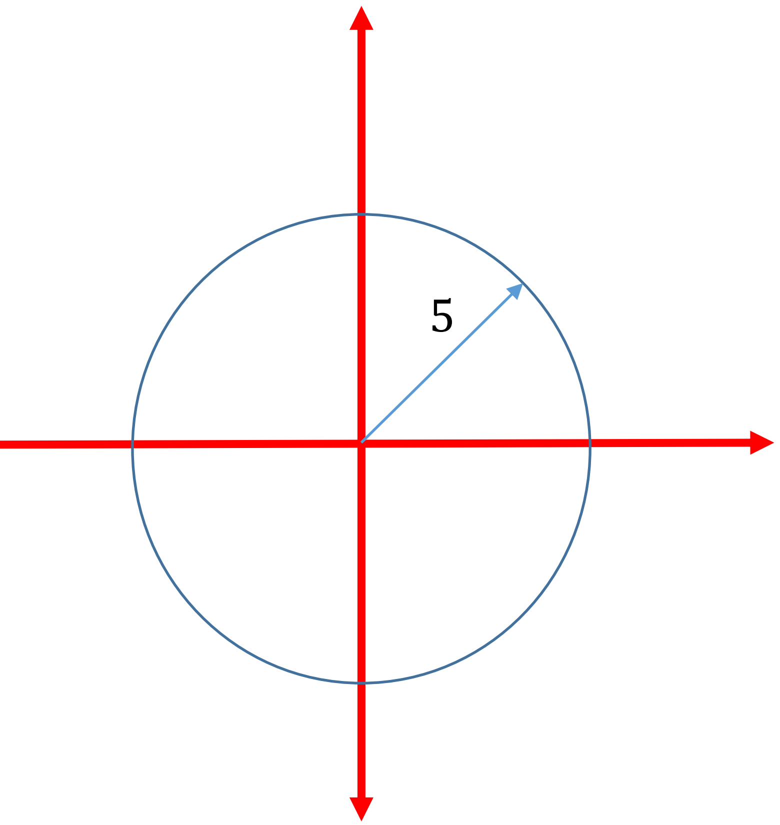
$R_3 = \{(x;y) \in \mathbb{R}^2 : x^2 + y^2 \leq 25\}$

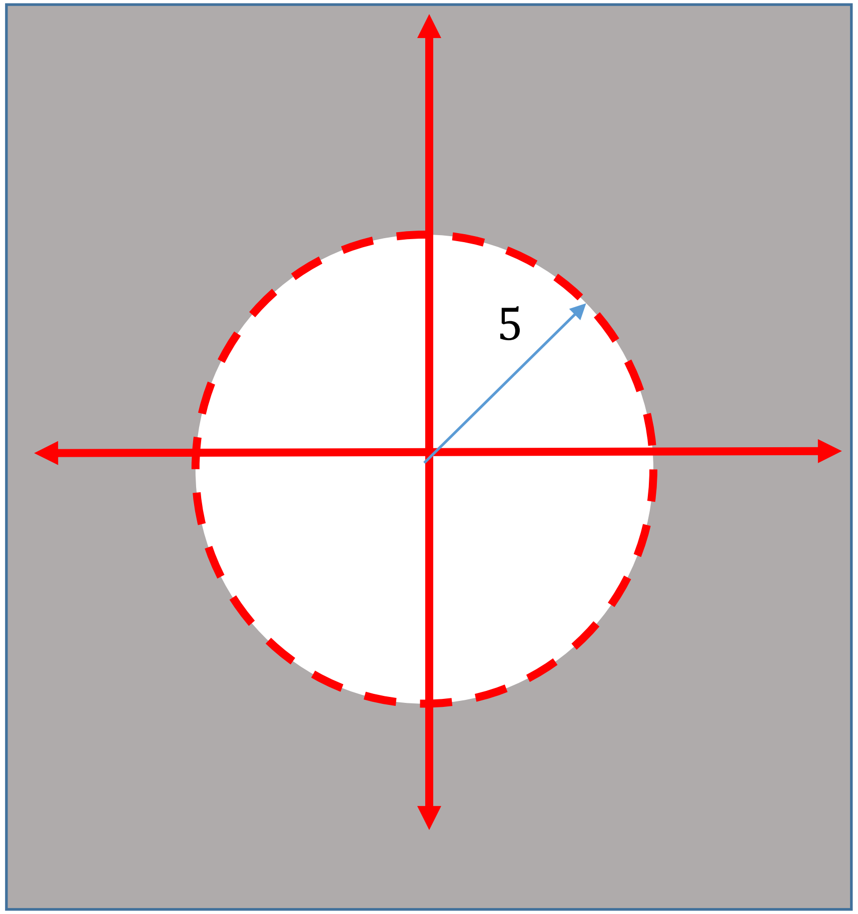


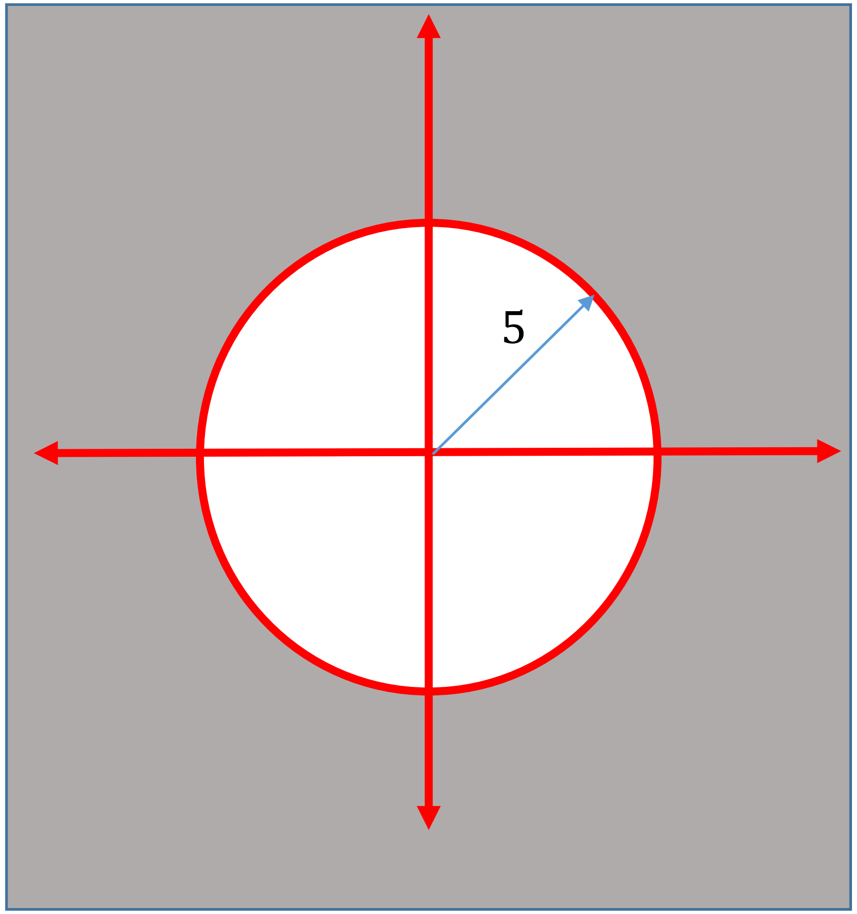
$R_1 = \{(x;y) \in \mathbb{R}^2 : x^2 + y^2 = 25\}$

$R_2 = \{(x;y) \in \mathbb{R}^2 : x^2 + y^2 > 25\}$

$R_3 = \{(x;y) \in \mathbb{R}^2 : x^2 + y^2 \geq 25\}$



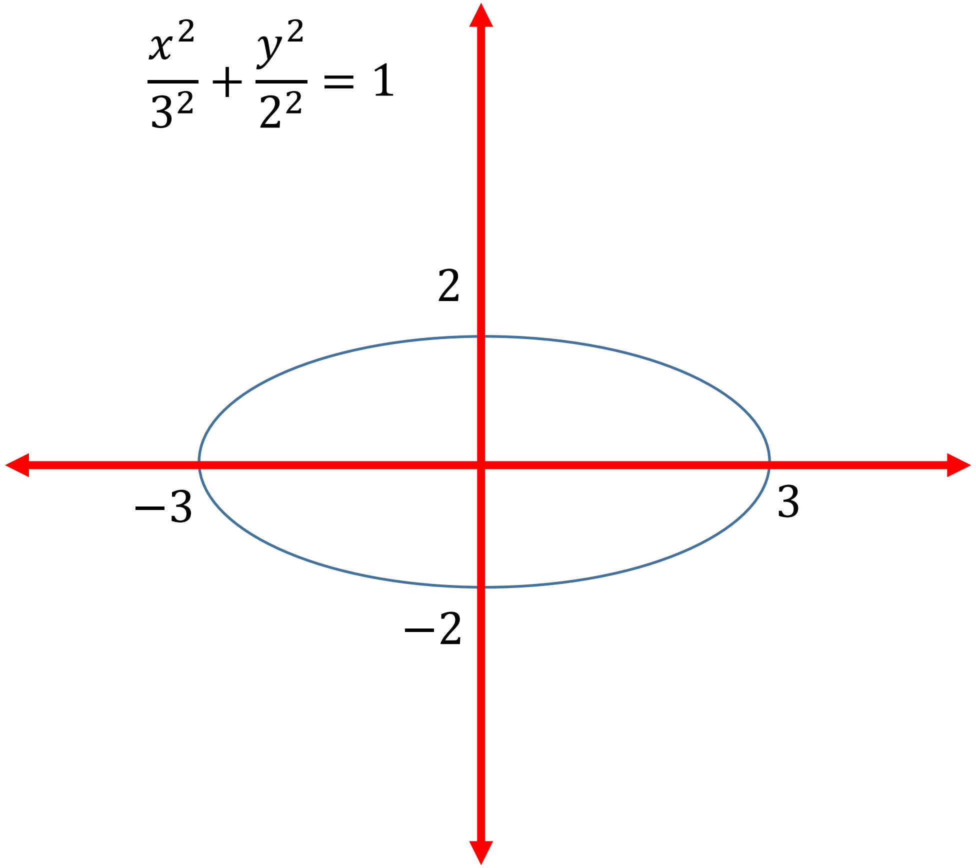




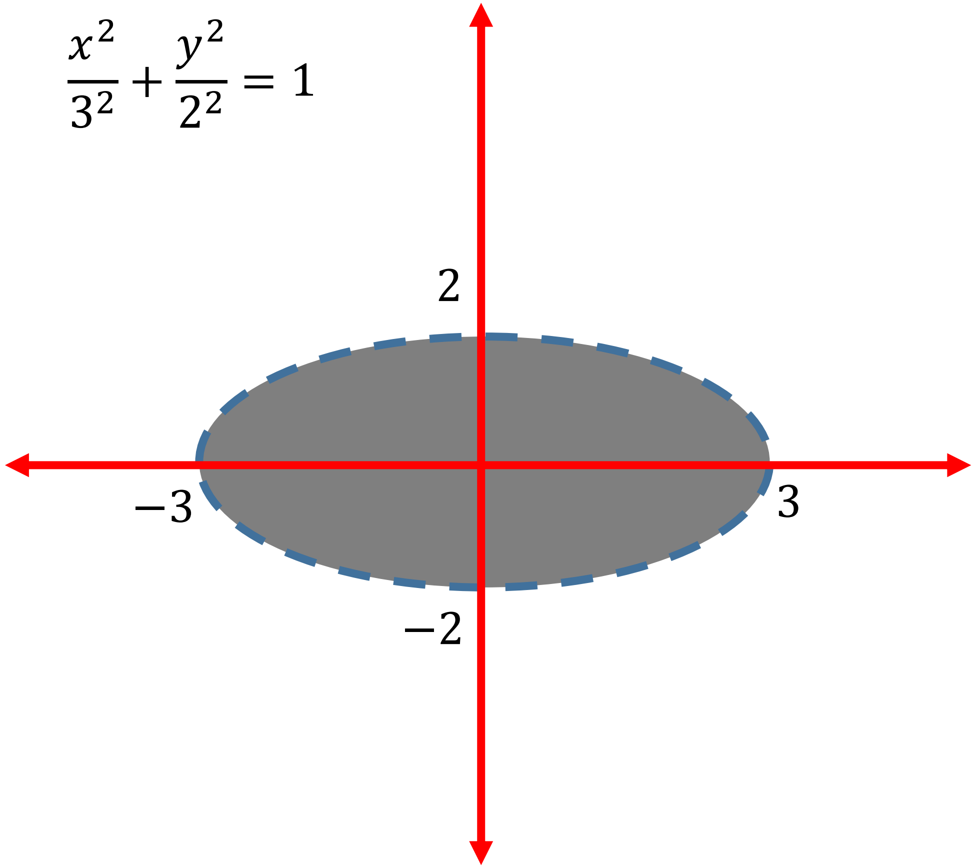
$$R_1 = \{(x;y) \in \mathbb{R}^2\colon\ \frac{x^2}{9} + \frac{y^2}{4} = 1\}$$

$$R_2 = \{(x;y) \in \mathbb{R}^2\colon\ \frac{x^2}{9} + \frac{y^2}{4} < 1\}$$

$$\frac{x^2}{3^2} + \frac{y^2}{2^2} = 1$$

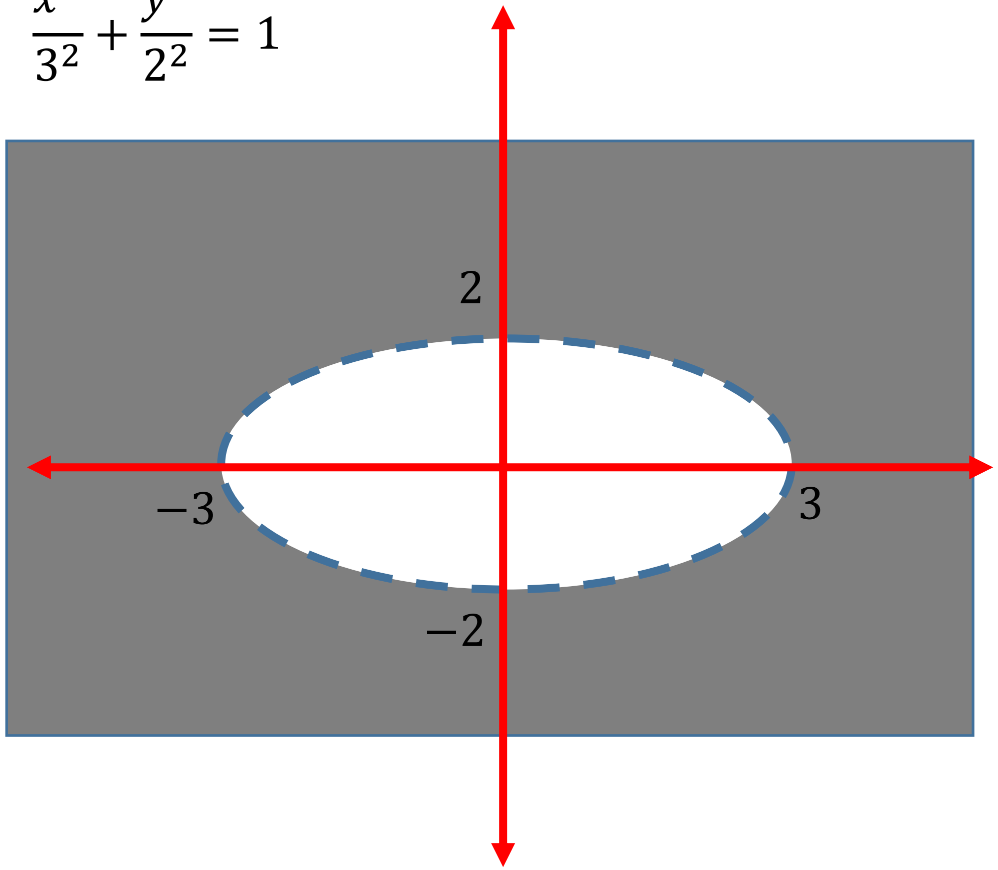


$$\frac{x^2}{3^2} + \frac{y^2}{2^2} = 1$$



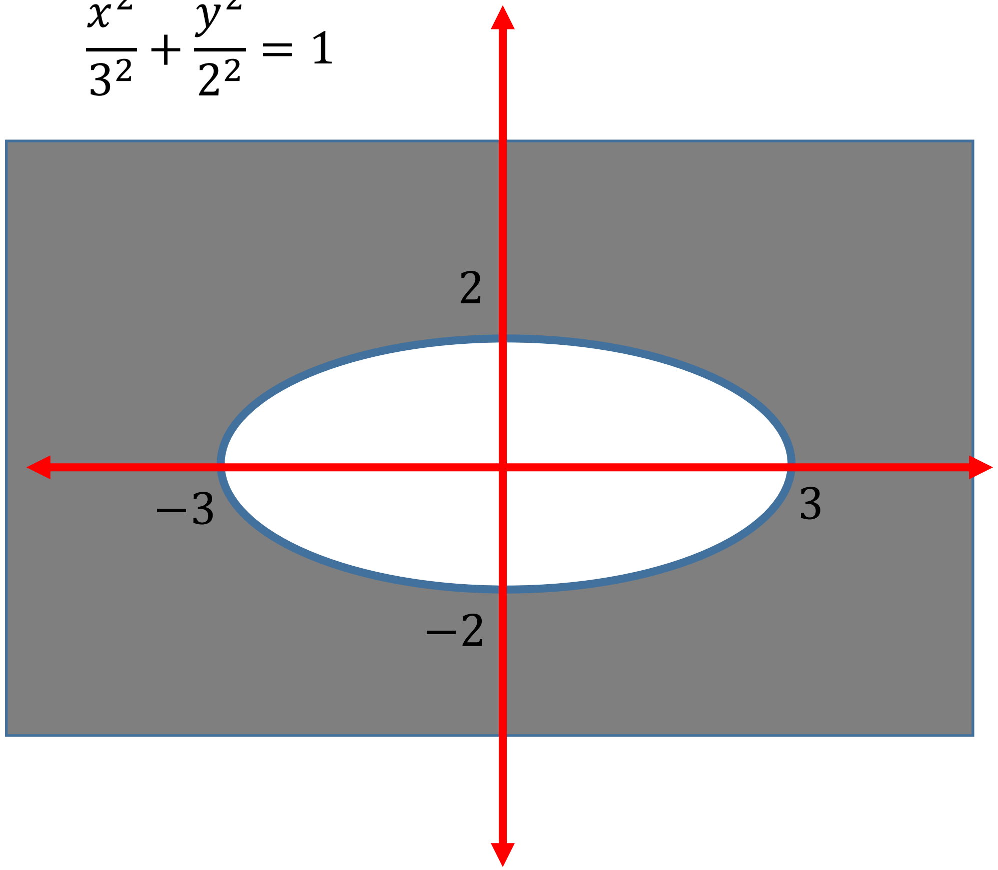
$$R_2 = \{(x;y) \in \mathbb{R}^2 \colon \frac{x^2}{9} + \frac{y^2}{4} > 1\}$$

$$\frac{x^2}{3^2} + \frac{y^2}{2^2} = 1$$



$$R_2 = \{(x;y) \in \mathbb{R}^2 \colon \frac{x^2}{9} + \frac{y^2}{4} \geq 1\}$$

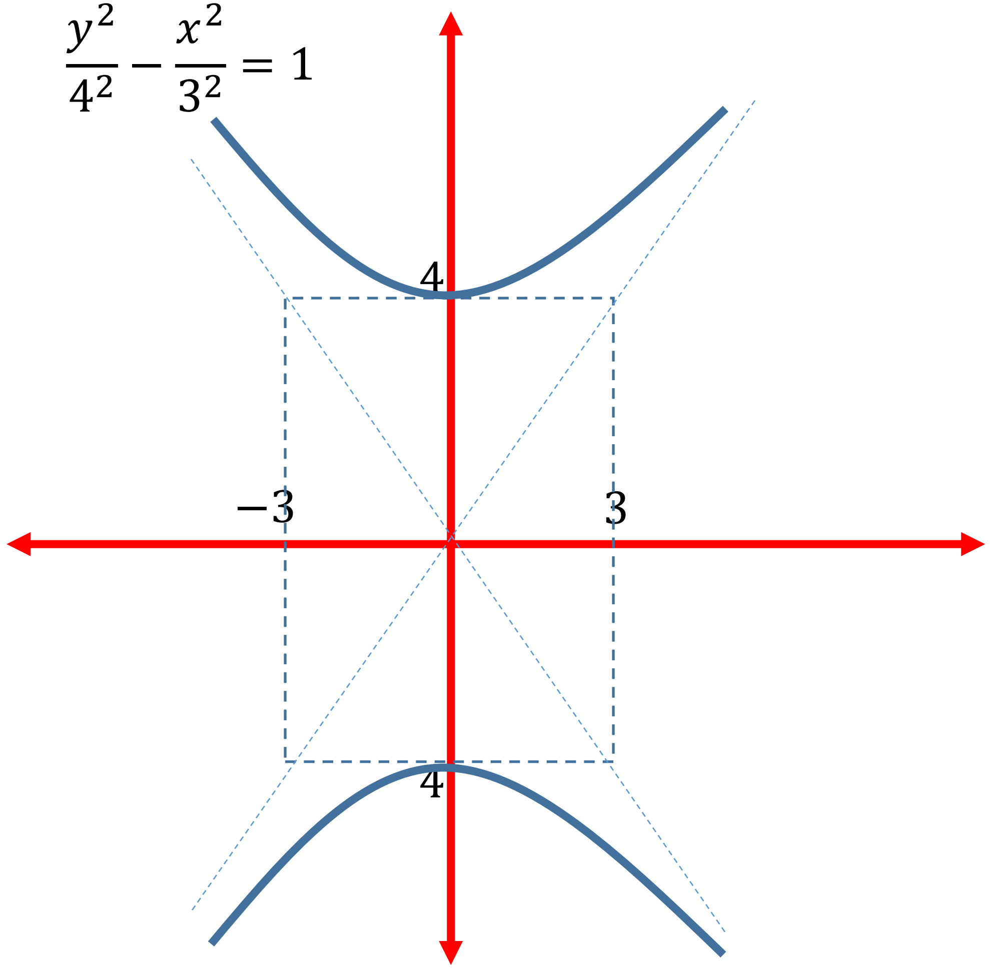
$$\frac{x^2}{3^2} + \frac{y^2}{2^2} = 1$$

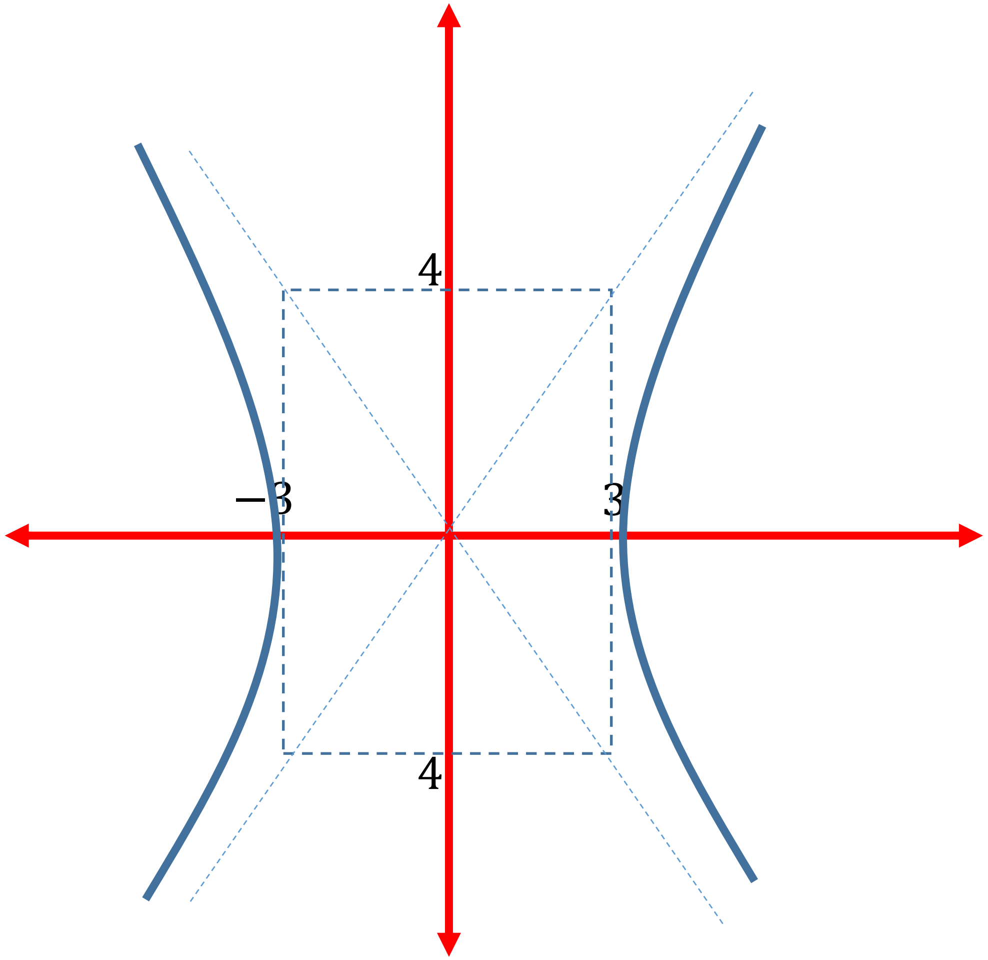


$$R_1 = \{(x;y) \in \mathbb{R}^2 : \frac{y^2}{16} - \frac{x^2}{9} = 1\}$$

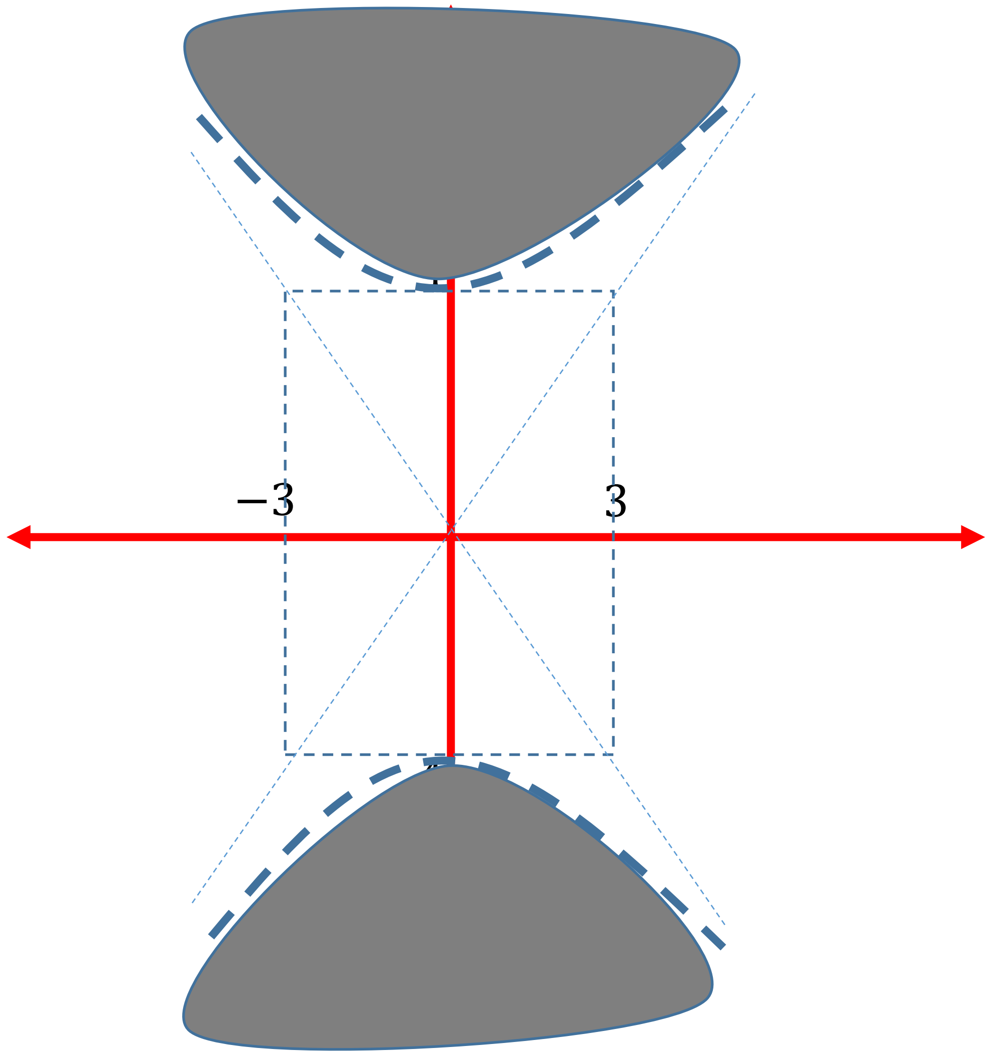
$$R_2 = \{(x;y) \in \mathbb{R}^2 : \frac{x^2}{9} - \frac{y^2}{16} = 1\}$$

$$\frac{y^2}{4^2} - \frac{x^2}{3^2} = 1$$

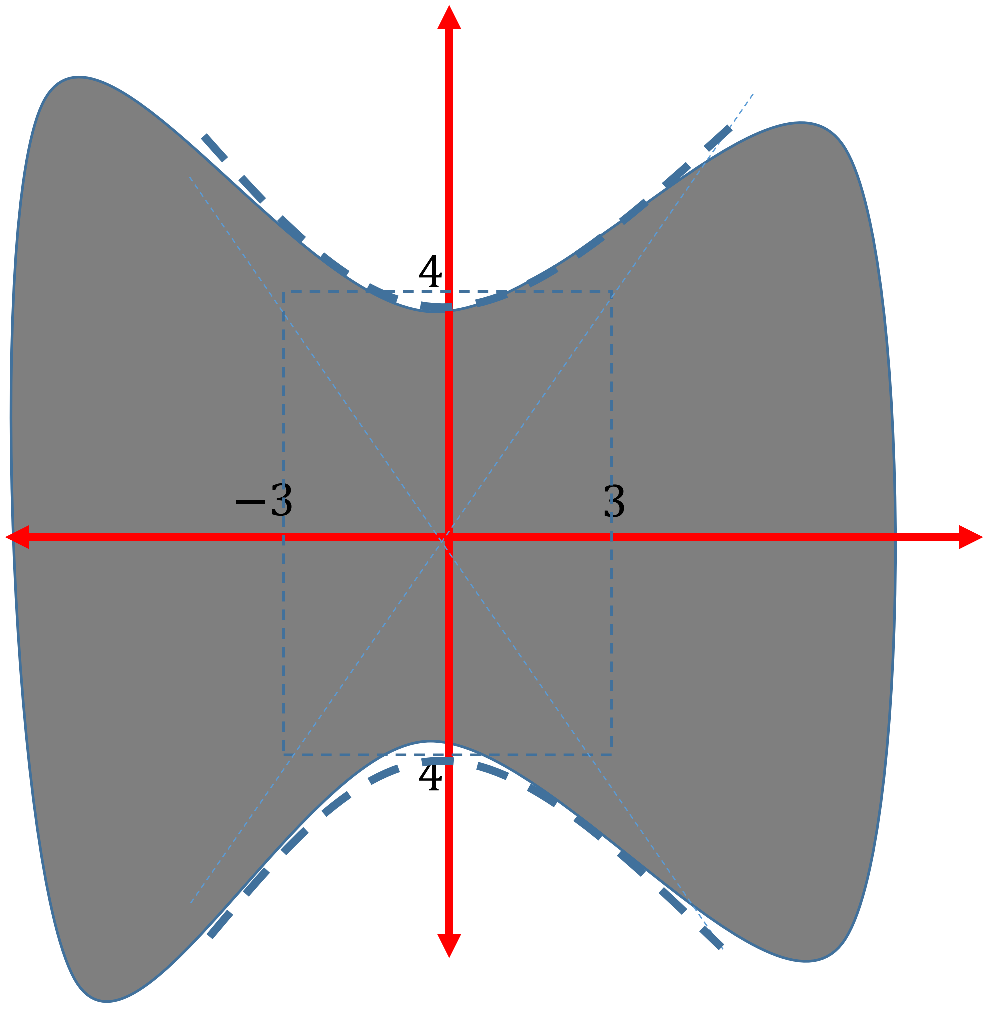




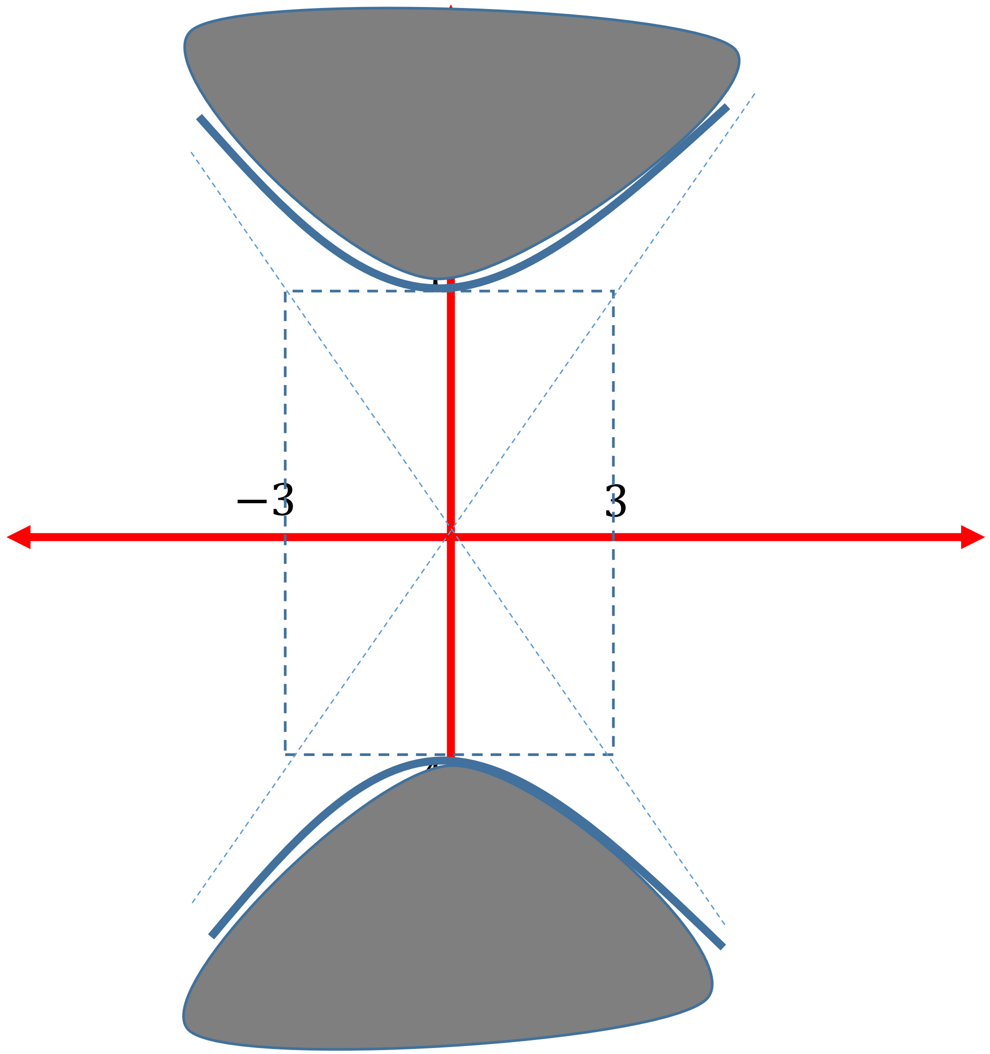
$$R_1 = \{(x;y) \in \mathbb{R}^2 : \frac{y^2}{16} - \frac{x^2}{9} > 1\}$$



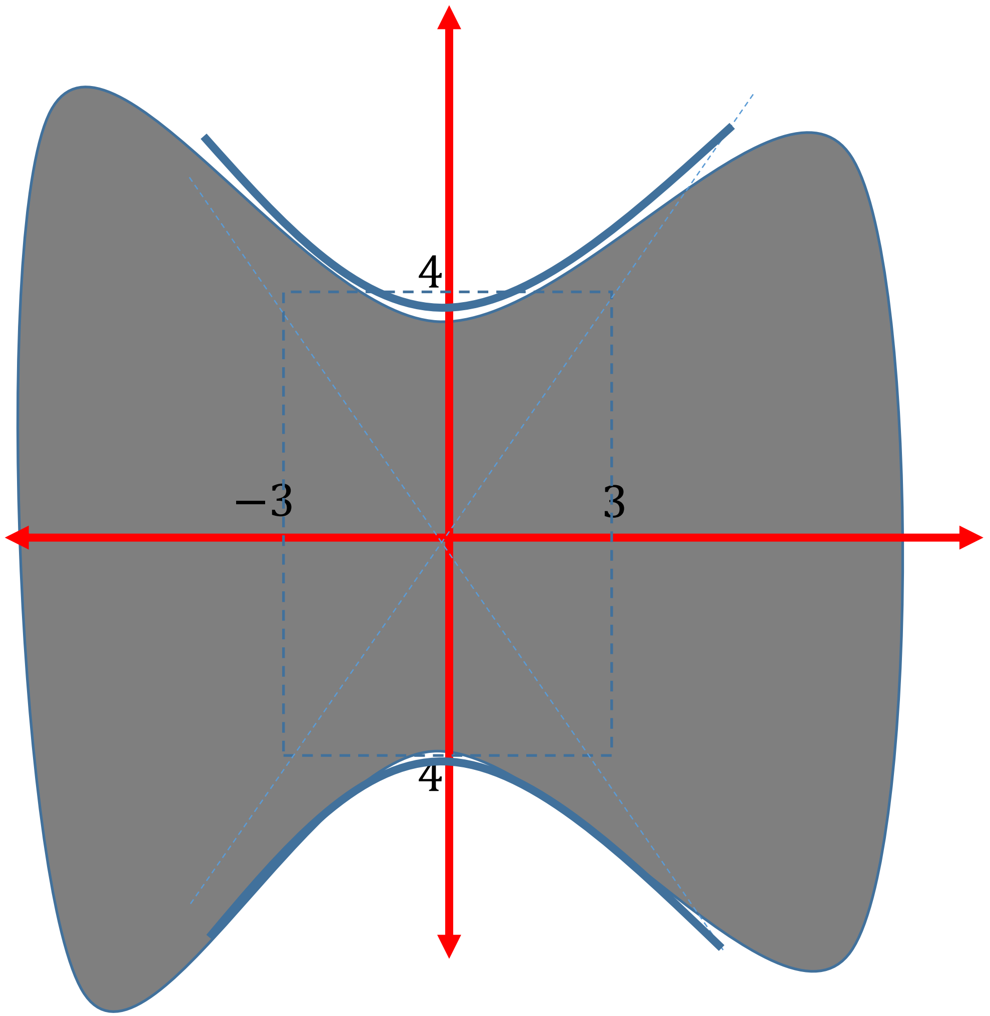
$$R_2 = \{(x;y) \in \mathbb{R}^2 : \frac{y^2}{16} - \frac{x^2}{9} < 1\}$$



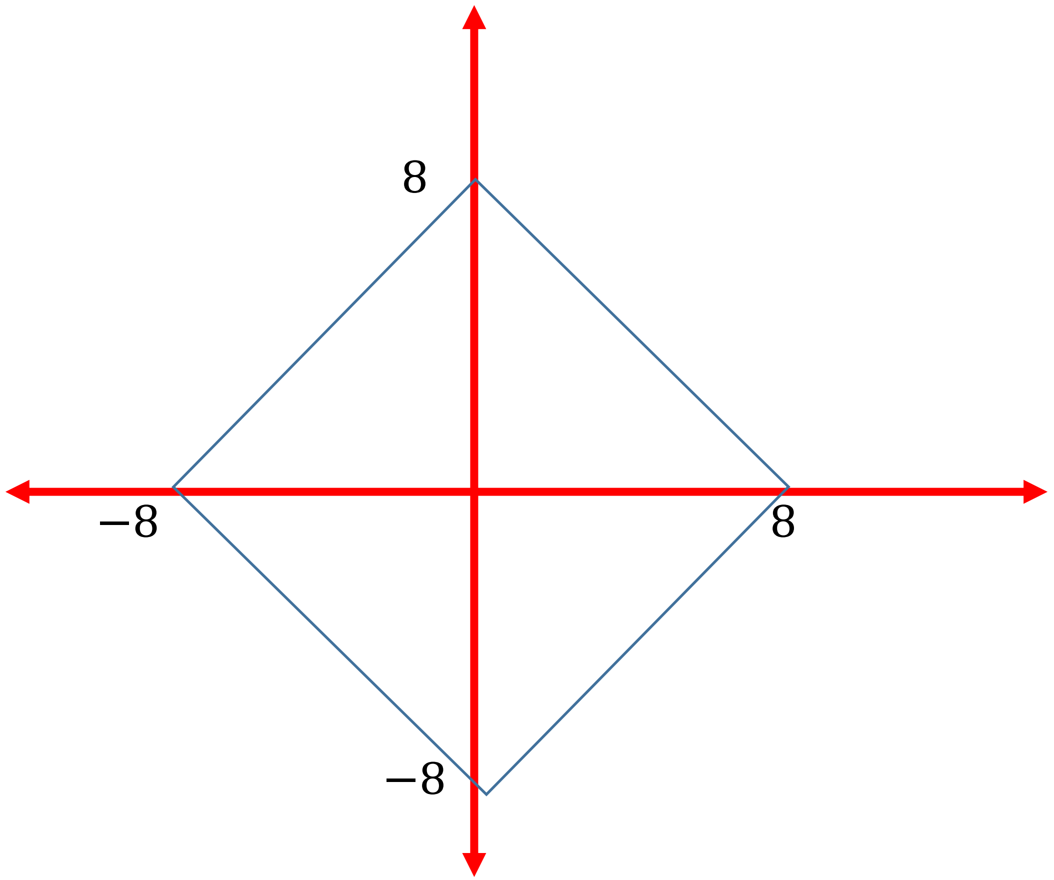
$$R_1 = \{(x;y) \in \mathbb{R}^2 : \frac{y^2}{16} - \frac{x^2}{9} \geq 1\}$$



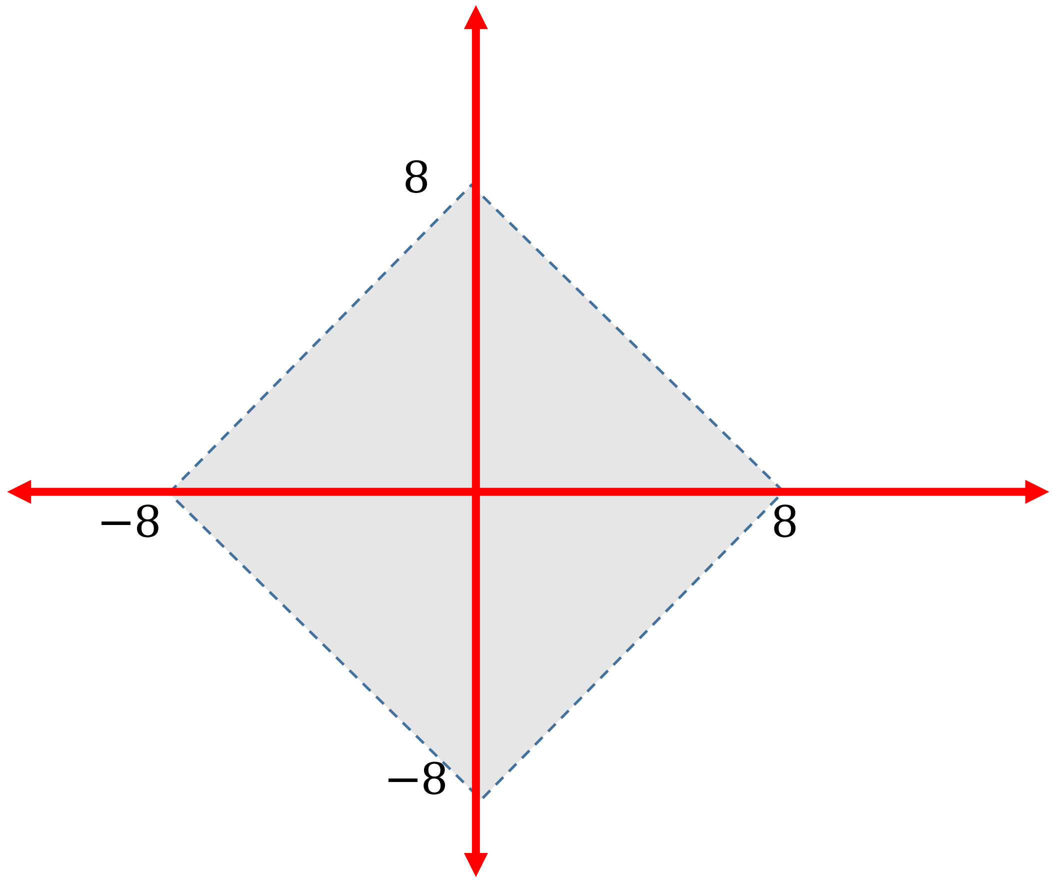
$$R_2 = \{(x;y) \in \mathbb{R}^2 : \frac{y^2}{16} - \frac{x^2}{9} \leq 1\}$$



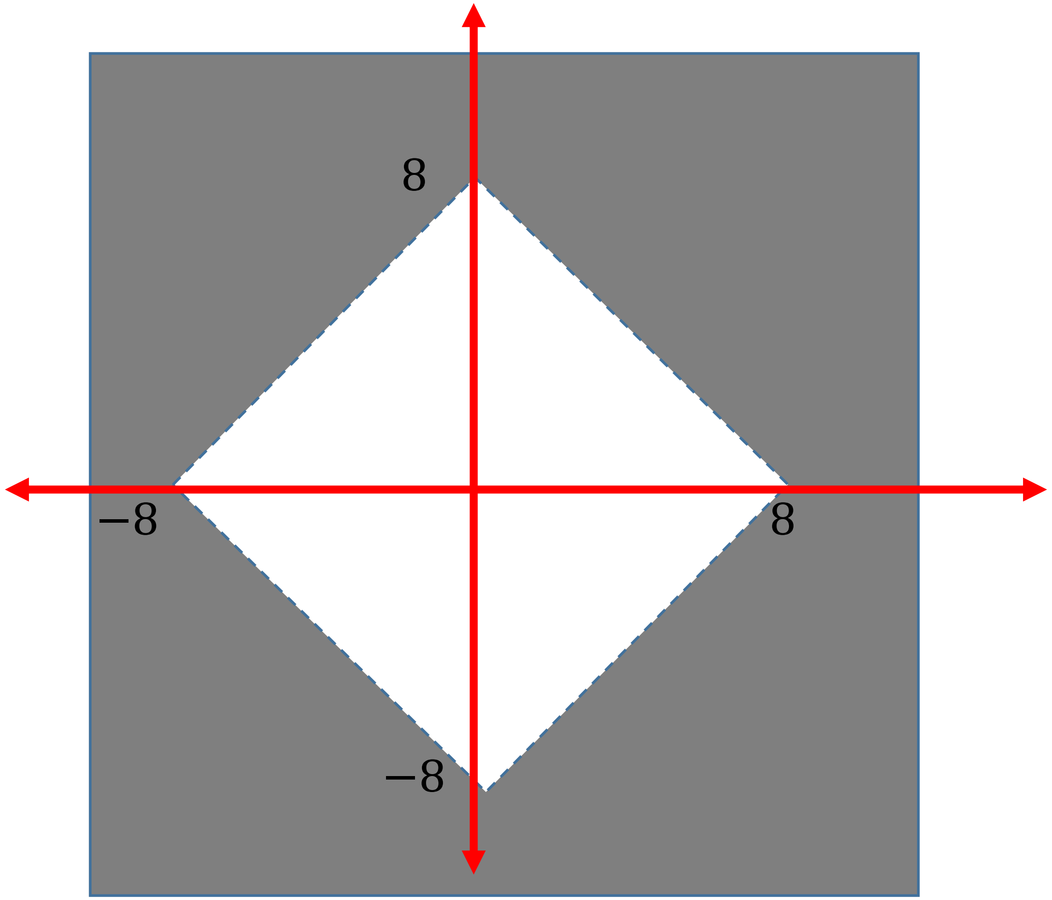
$$R_1 = \{(x; y) \in \mathbb{R}^2 : |x| + |y| = 8\}$$



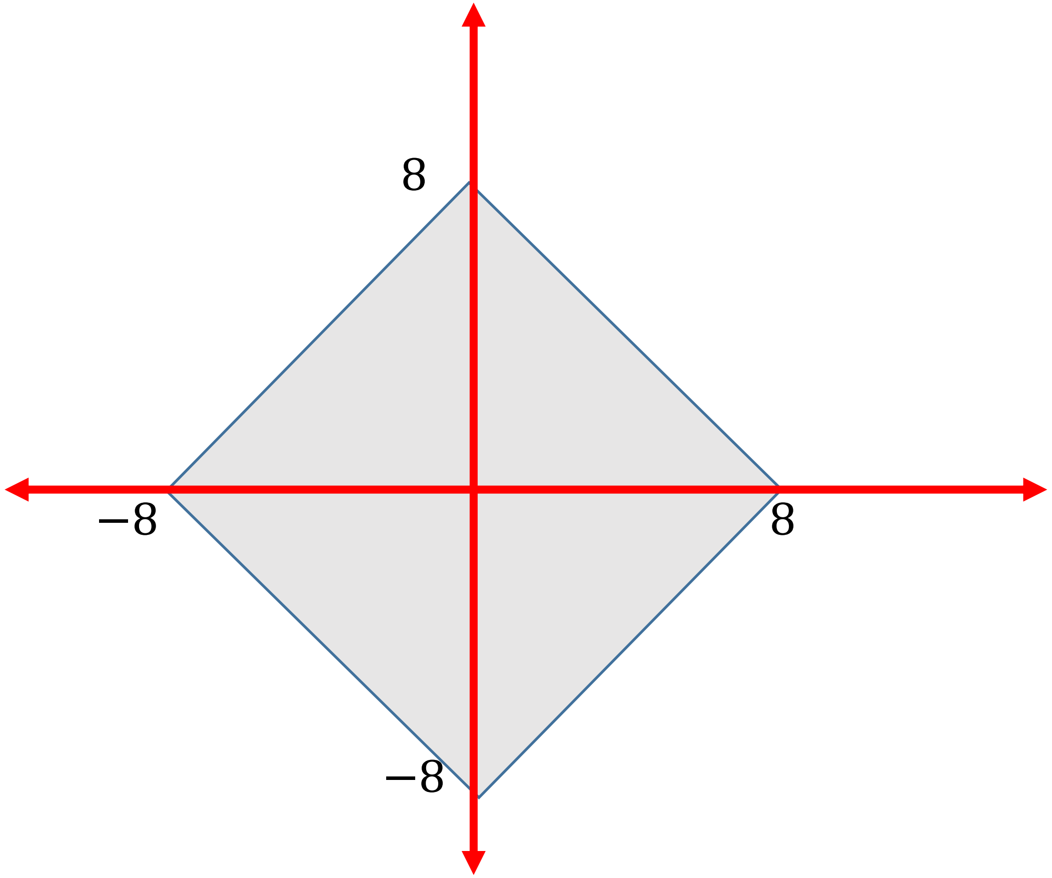
$$R_1 = \{(x; y) \in \mathbb{R}^2 : |x| + |y| < 8\}$$



$R_1 = \{(x; y) \in \mathbb{R}^2 : |x| + |y| > 8\}$



$R_1 = \{(x; y) \in \mathbb{R}^2 : |x| + |y| \leq 8\}$



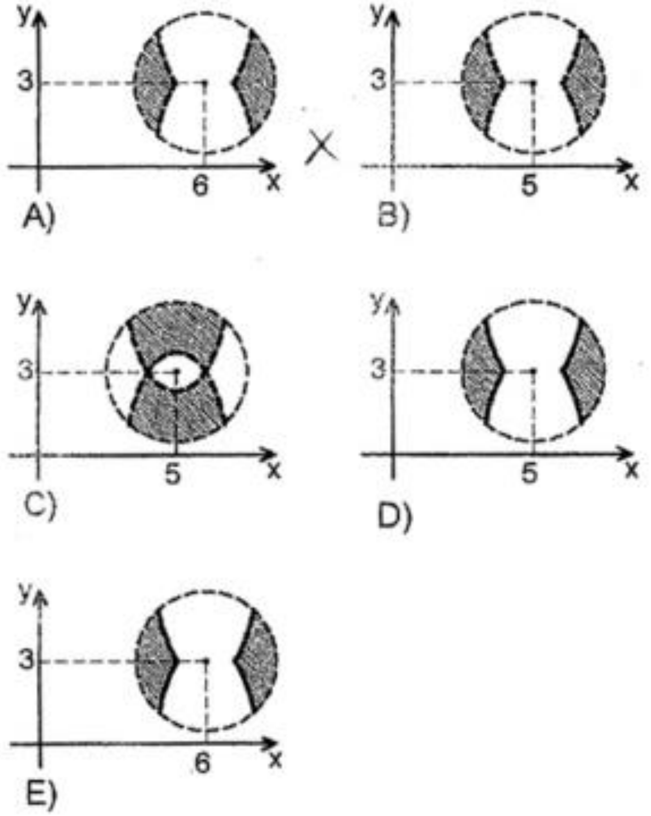
19. Dado el sistema de inecuaciones

$x^2+y^2-10x-6y<-30$,

$y-x^2+10x<27$,

$10x-x^2-y<21$.

Señale el gráfico más próximo al conjunto solución del sistema anterior.



$$x^2+y^2-10x-6y+30<0$$

$$(x-5)^2+(y-3)^2<4$$

Circunferencia con centro en $(5;3)$

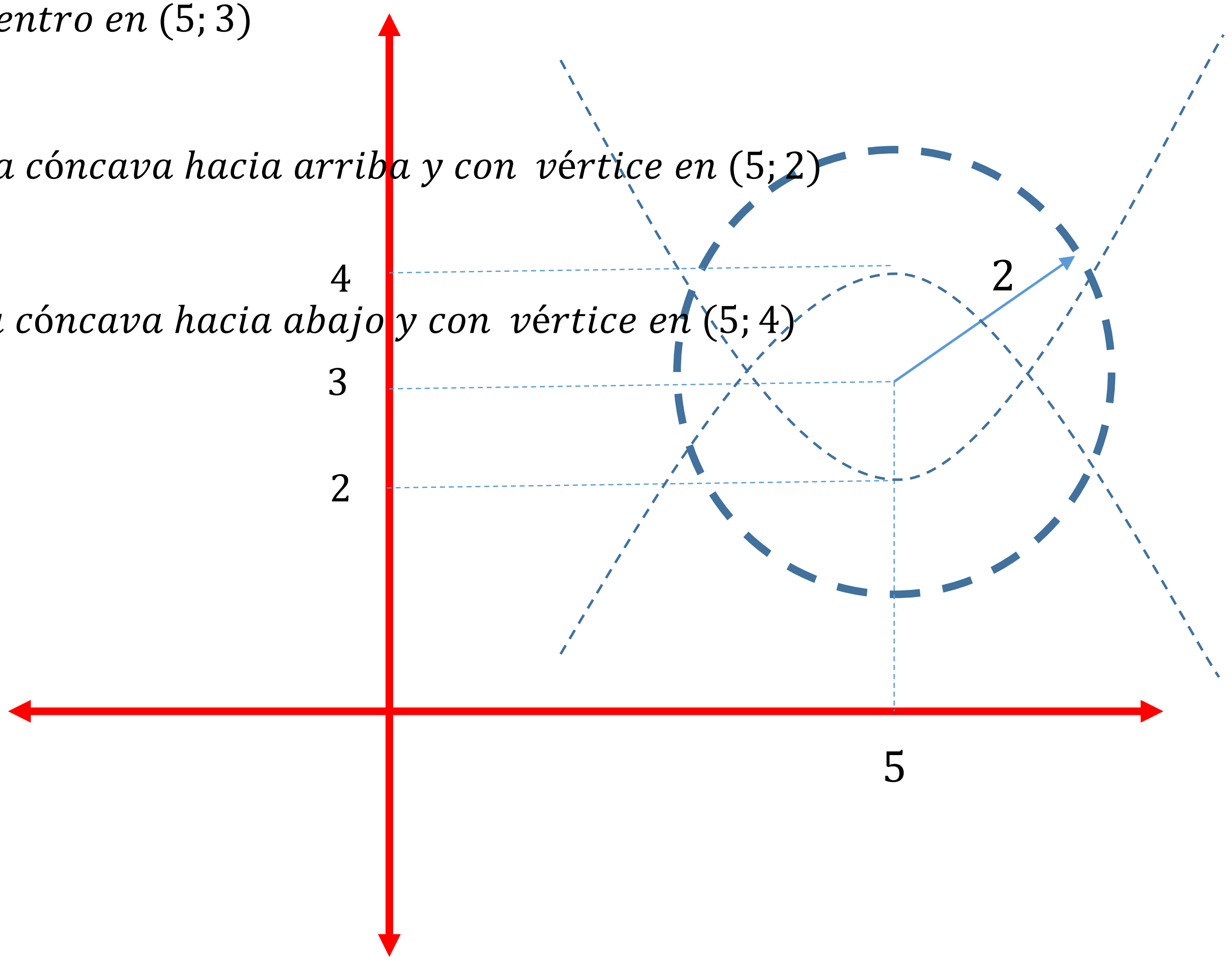
$$y<(x-5)^2+2$$

Debajo de la parábola cóncava hacia arriba y con vértice en $(5;2)$

$$y>4-(x-5)^2$$

Encima de la parábola cóncava hacia abajo y con vértice en $(5;4)$

CLAVE B



20. Sean $\|(x,\ y)\|_1 = |x| + |y|$.

$\|(x,\ y)\|_2 = \text{máx}\{|x|, |y|\}$ para $(x, y) \in \mathbb{R}^2$

Calcule el área de la región C, donde

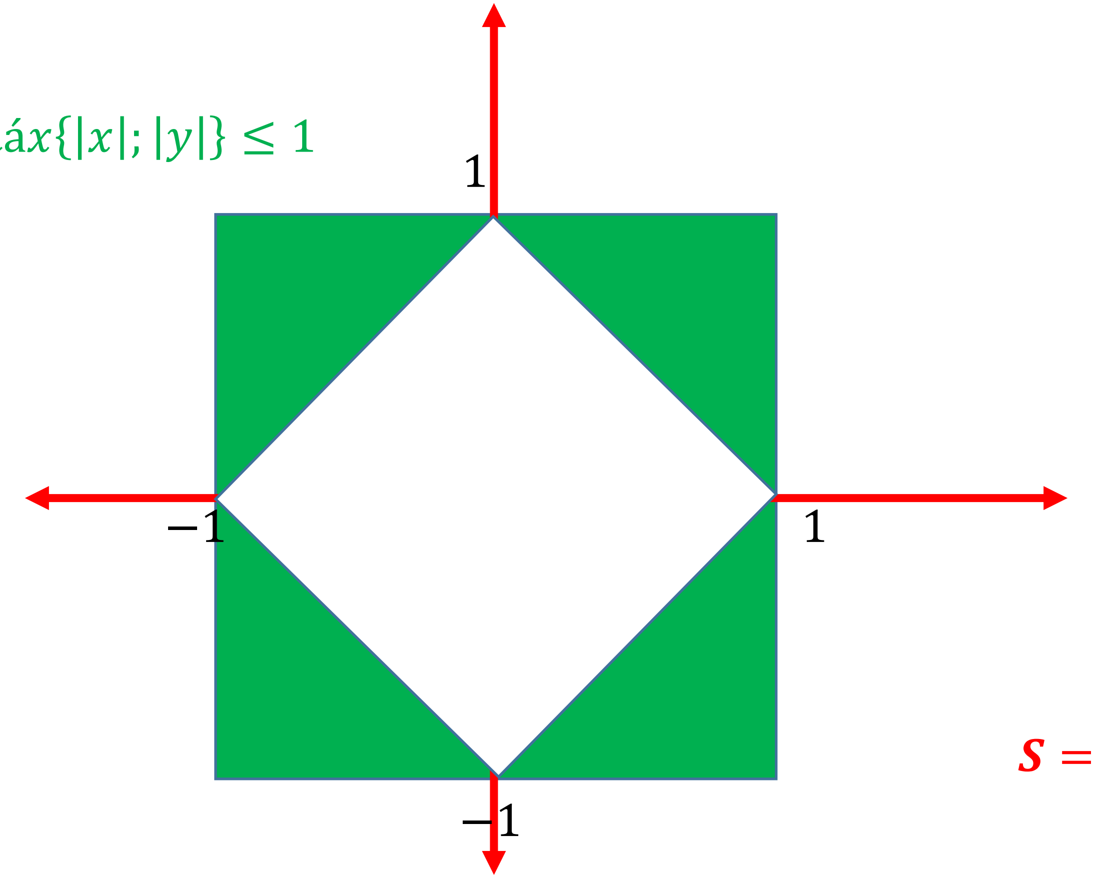
$C = \{(x,y) : \|(x,y)\|_2 \leq 1 \text{ y } \|(x,y)\|_1 \geq 1\}$

A) 0
B) 1
C) $\sqrt{2}$
D) 2
E) $2\sqrt{2}$

$$C = \{(x:y):\ \|(x:y)\|_2 \leq 1 \quad \wedge \|(x:y)\|_1 \geq 1\}$$

$$\boxed{C = \{(x:y):\ máx\{|x|;|y|\} \leq 1 \quad \wedge |x| + |y| \geq 1\}}$$

$máx\{|x|; |y|\} \leq 1$



$$S = 4\left(\frac{1x1}{2}\right) = 2$$

PROBLEMAS RESUELTOS

PROBLEMA

04.- Sea el conjunto:

$$A = \{1; 2; 3\}$$

Y sean las relaciones R, S y T definidas en A; donde R, S y T son reflexiva, simétrica y transitiva, respectivamente, si:

$$R = \{(1; a), (2; 3), (2; b), (3; c)\}$$

$$S = \{(1; 3), (e; d)\}$$

$$T = \{(1; 2), (2; 3), (f; g)\}$$

SOLUCIÓN

$$Reflexiva: \quad \forall x \in A \; se\ cumple\ que\ (x; x) \in A$$

$$Ya\ que\ R\ es\ reflexiva\ y\ R = \{(1; a)(2; 3), (2; b), (3; c)\} \quad a = 1 \quad b = 2 \quad c = 3$$

$$Simétrica: \quad \forall (x; y) \in R\ se\ cumple\ que\ (y: x) \in R$$

$$Ya\ que\ S\ es\ simétrica\ y\ S = \{(1; 3)(e; d)\} \qquad entonces\ e = 3 \;\; y\ d = 1$$

$$Transitiva: \quad \forall (x; y) \wedge (y; z) \in R\ se\ cumple\ que\ (x; z) \in R$$

$$Ya\ que\ T\ es\ transitiva\ y\ S = \{(1; 2). (2; 3), (f; g)\} \quad entonces\ f = 1 \;\; g = 3$$

$$Sea\ A\ el\ conjunto\ \{4; 5; 7; 8\}$$

$$y\ sea\ R = \{(x; y) \in A^2\ tal\ que\ y \geq x\}$$

$$entonces\ R = \{(4;4), (4;5), (4;7), (4;8), (5;5), (5;7), (5;8), (7;7), (7;8), (8;8)\}$$

$¿R\ es\ reflexiva?$

$Reflexiva$: $\forall x \in A\ se\ cumple\ que\ (x; x) \in R$

$Ya\ que\ (4;4), (5;5), (7;7), (8;8) \in R\ entonces\ R\ es\ reflexiva$

$¿R\ es\ simétrica?$

$Simétrica$: $\forall (x; y) \in R\ se\ cumple\ que\ (y: x) \in R$

$No\ es\ simétrica$

$¿R\ es\ transitiva?$

$Transitiva$: $\forall (x; y) \wedge (y; z) \in R\ se\ cumple\ que\ (x; z) \in R$

$Sí\ es\ transitiva$

$Si\ R\ es\ reflexiva.\ simétrica\ y\ transitiva\ entonces\ entonces\ se\ le\ llama$ $Relación\ de\ Equivalencia$

$$Sea\ \mathbb{Z}\ el\ conjunto\ de\ los\ números\ enteros$$
$$y\ sea\ R = \{(x;y) \in \mathbb{Z}^2\ tal\ que\ x - y\ sea\ múltiplo\ de\ 3\}$$

$$entonces\ R = \{\ldots.(4;1),(8;5),(10;7),(11;8),(5;2),(9;9),(2;2).(7;4),(10;4),(2;5)\ldots\}$$

$¿R\ es\ reflexiva?$

$Reflexiva\colon\quad \forall x \in A\ se\ cumple\ que\ (x;x) \in R$

$Sí\ es\ reflexiva$

$¿R\ es\ simétrica?$

$Simétrica\colon\quad \forall(x;y) \in R\ se\ cumple\ que\ (y\colon x) \in R$

$Sí\ es\ simétrica$

$¿R\ es\ transitiva?$

$Transitiva\colon\quad \forall(x;y) \wedge (y;z) \in R\ se\ cumple\ que\ (x;z) \in R$

$Sí\ es\ transitiva$

$Si\ R\ es\ reflexiva.simétrica\ y\ transitiva\ entonces\ entonces\ se\ le\ llama\ Relación\ de\ Equivalencia$

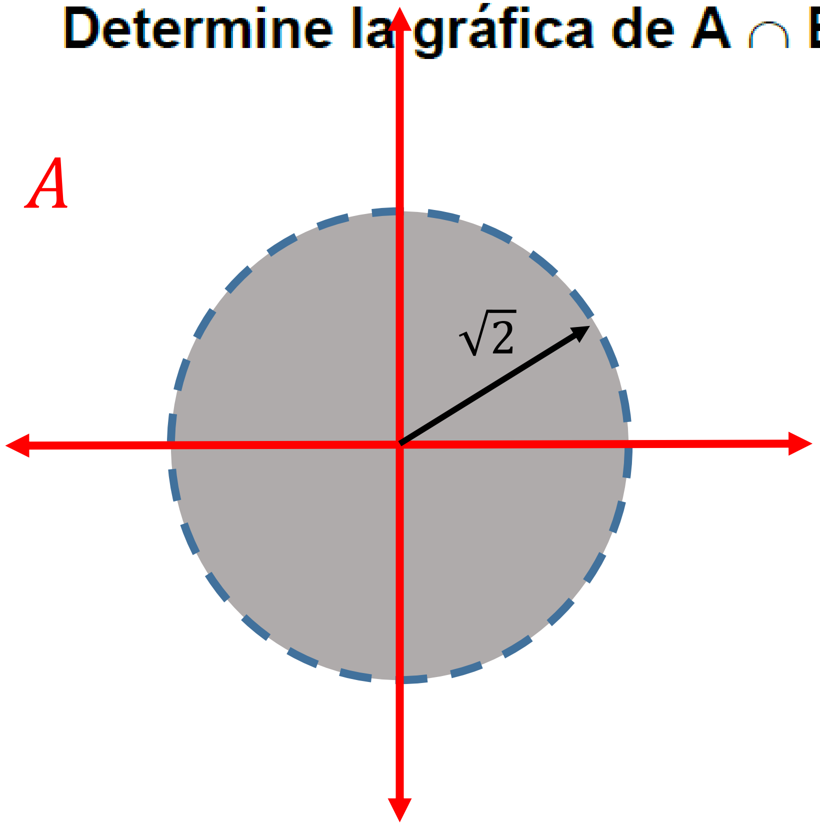
01. Si:

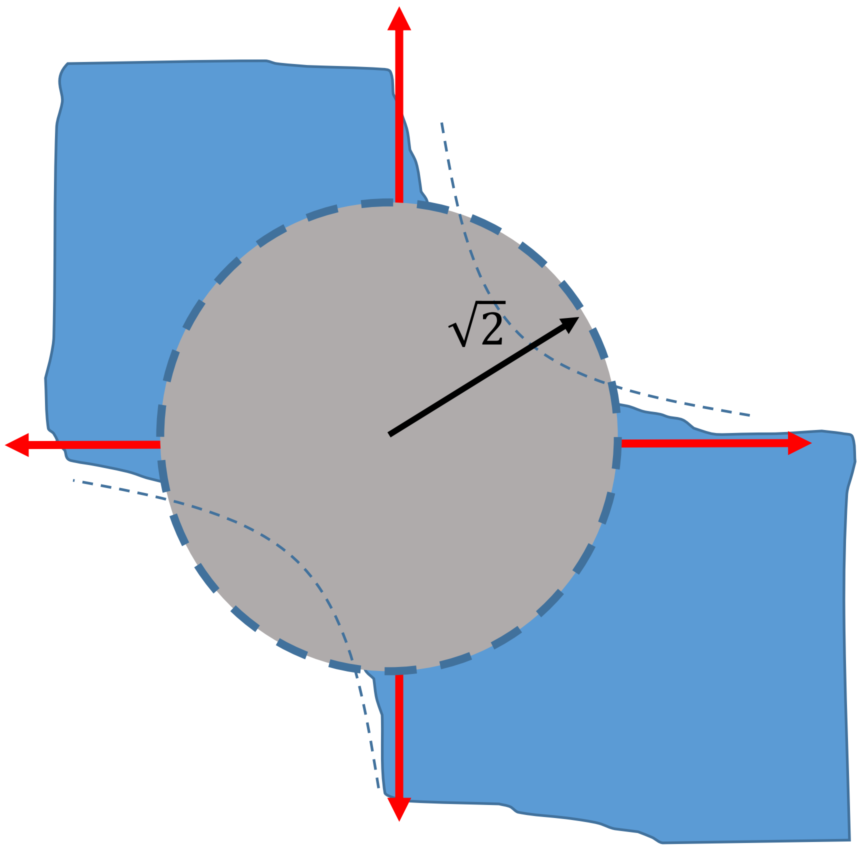
$$A = \{(x, y) \in R^2 / x^2 + y^2 < 2\}$$

$$B = \{(x, y) \in R^2 / xy < 1\}$$

Determine la gráfica de $A \cap B$.

A



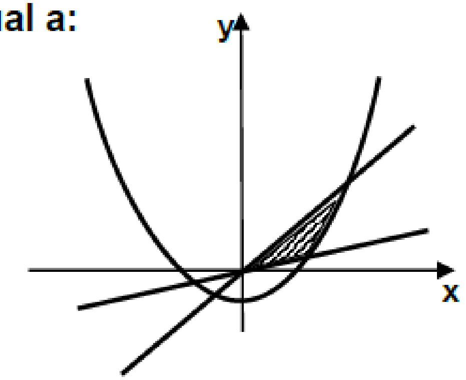


02. Dados los conjuntos

$A = \{(x, y) \in \mathbb{R}^2 \,/\, y/4 \leq x/2 \leq y\}$

$B = \{(x, y) \in \mathbb{R}^2 \,/\, y \leq (x - 1)(x + 1)\}$.

La región sombreada (ver figura) expresado en términos de A y B es igual a:

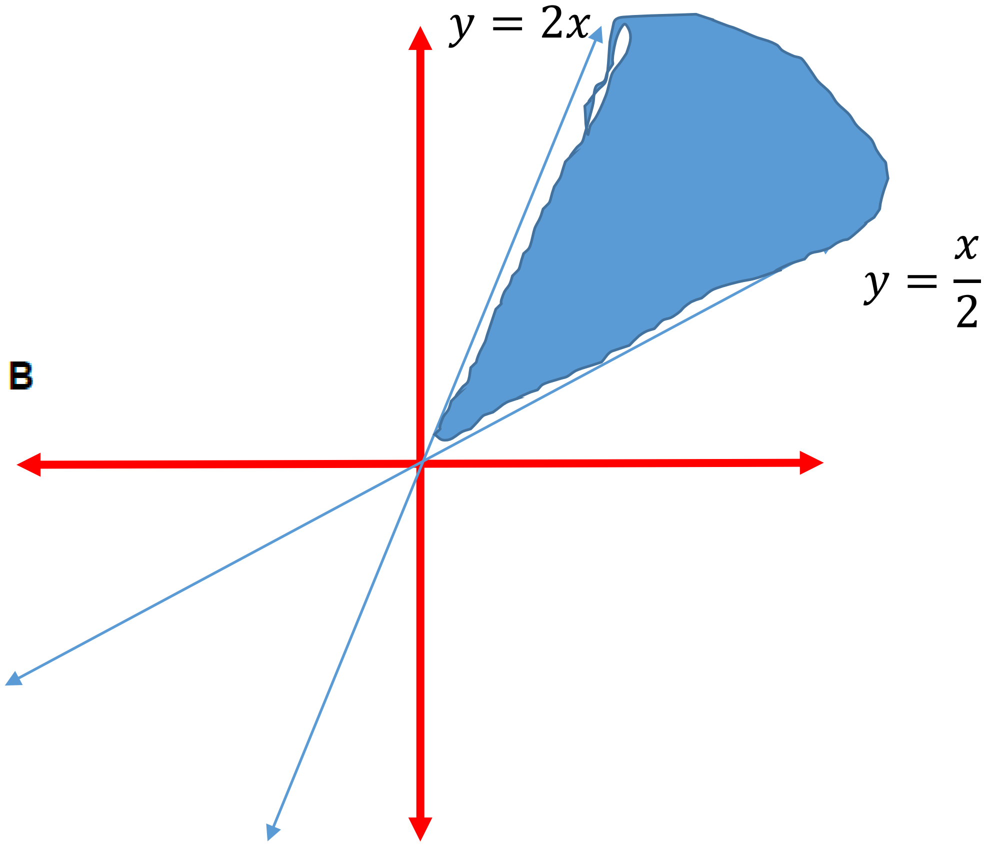


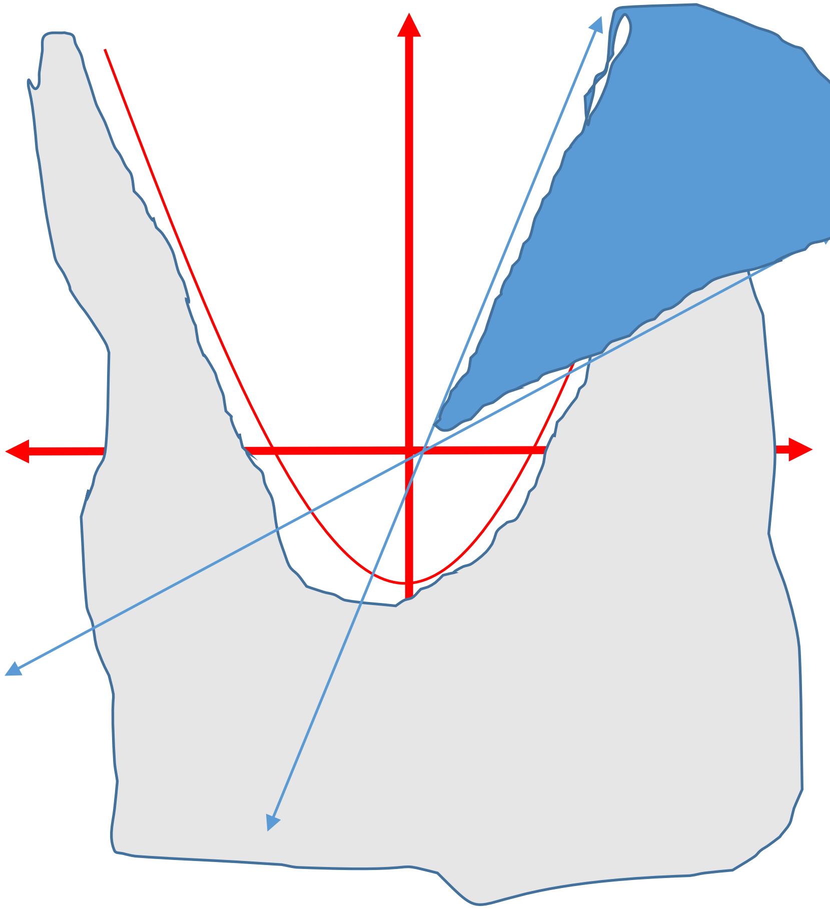
A) B – A B) A ∩ B C) A – B

D) $(A - B)^C$ E) $A \cup B^C$

$$\frac{y}{4} \leq \frac{x}{2} \leq y$$

$$\frac{y}{4} \leq \frac{x}{2} \quad \textit{de donde } y \leq 2x \qquad \textit{y además } \frac{x}{2} \leq y$$





$S = \{(x; y) \in \mathbb{R}^2 / |y|-|x| > 1 \wedge |x| \leq 2 \wedge 16y^2 + 25x^2 < 400\}$

$|y| > |x| + 1$